Math 120E (Midterm)

!!!!! READ READ READ !!!

Instructions:

- **Show your work; no credit for answers only.**

- **Write your name on the exam.**

- **If you are using a graphing calculator, "zooming in" to find a value on a function graph, using "solver" programs, and so on, will not be sufficient justification for any answer on this exam. You are free to use the calculator to check yourself. When in doubt, ask a question by raising your hand. I will come around to help as soon as possible.**

SCORE (40 possible):

Problem #3 (15) ▬▬▬▬▬▬▬▬

Problem #4 (25) ▬▬▬▬▬▬▬▬

TOTAL ▬▬▬▬▬▬▬

TWO DAY TOTAL ▬▬▬▬▬▬▬

Problem 3 (15 points):

(a) (7 points) Let $w(t) = t^2 - 3t + 2$. Simplify the expression

$$\frac{w(t+h) - w(t)}{h}$$

as far as possible. (There should NOT be an h left in the denominator.)

(b) (8 points) $h(x) = 2x - 1$, $g(x) = 4x^2 + x + 5$

   i. What are the domain and range of the function $y = \sin(h(x))$?

   ii. What are the domain and range of the function $y = h(\sin(x))$?

   iii. Write $g(x)$ in vertex form. That is, rewrite it in the form $a(x - h)^2 + k$, for specific values of a, h, and k.

Problem 4 (25 points): Carl and Joseph are training for the Seattle Half-Marathon, held over Thanksgiving weekend. They are running counter-clockwise on the track pictured below. Each of the two ends of the track is a semi-circle. The inner semi-circle has radius 50 feet and the outer semi-circle has radius 52 feet. The centers of the circles lie on the x-axis, 200 feet apart. Joseph and Carl start running at the same time, at the positions shown in the picture.

Carl starts at (0,52)
(200,52)
Joseph starts at (0,50)
(0,0)
(200,0)
(200, -52)
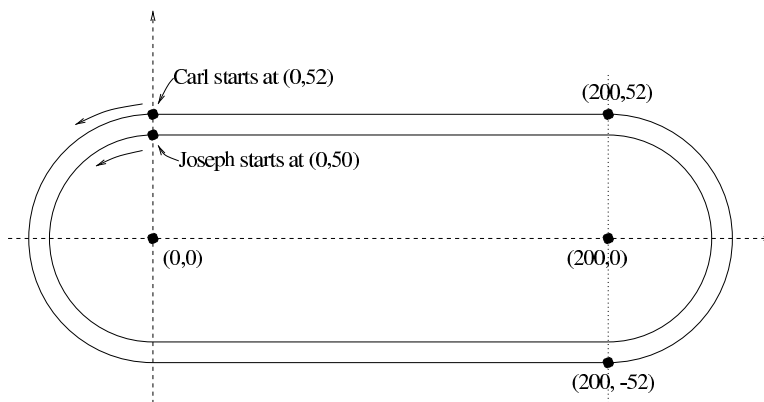
This drawing is not to scale.

(a) (4 points) Carl runs on the outer track. It takes him 1.5 minutes to run each lap. Assume Carl runs at a constant linear speed. What is Carl's linear speed, in ft/s?

(b) (4 points) Joseph runs on the inner track. He runs at a constant 7.5 ft/s. How long does it take Joseph to run one lap?

(c) (3 points) When does Carl reach the point $(200, 52)$?

(d) (6 points) When Carl reaches the point $(200, 52)$, what are Joseph's coordinates?

(e) (8 points) Let $x_c(t)$ be the x-coordinate of Carl t seconds after he starts running. Give the multi-part formula for $x_c(t)$ from the time Carl starts running until he reaches the point $(200, -52)$. Clearly label the domain for each part of the function.