Math 120 Final Examination Autumn, 1997

Print Your Name	Write Your Signature
Student ID Number	**Quiz Section**
Professor's Name	**TA's Name**

!!! READ...INSTRUCTIONS...READ !!!

1. The exam contains 7 problems. Your exam packet should contain 10 pages. The value of each problem is clearly indicated.
2. Place a box around $\boxed{\text{YOUR ANSWER}}$ whenever appropriate.
3. All work must be shown. Show enough work so that the grader can tell how you obtained your answer. No credit will be assigned for answers only. Use of graphing calculator is not sufficient justification for any answers on the exam. If in doubt, ask for clarification.
4. **The exam ends promptly at 7:50pm.** If you have a question, please raise your hand and someone will assist you as soon as possible.
5. Good Luck!

Problem	Total Possible	Score
1	16	
2	14	
3	14	
4	10	
5	18	
6	10	
7	18	
TOTAL	100	

Problem 1 (16 points) Computational skills. Show your work on each problem.

(a) (4 pts.) Let $f(x) = x^2 - 2x$.

(i) (2 pts.) Find $f(3+h)$ and simplify.

(ii) (2 pts.) Find $\dfrac{f(3+h) - f(3)}{h}$ and simplify; the simplified result is not a fraction.

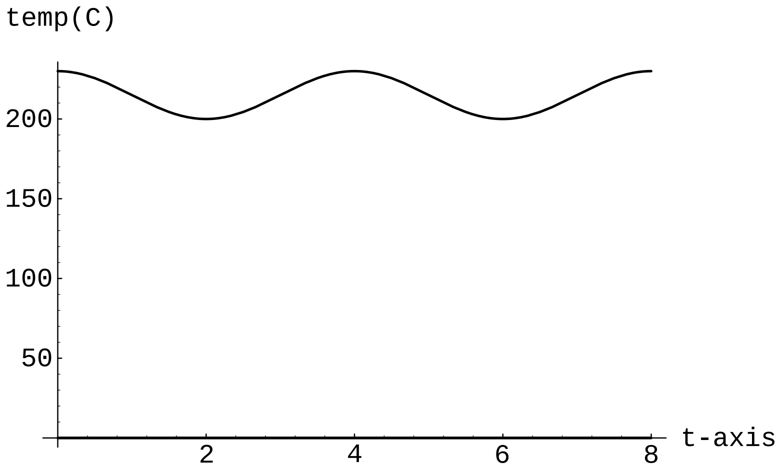
(b) (4 pts.) Let $f(x) = x^2 - 2x$ and $g(x) = \cos(\frac{\pi}{3}x)$. Compute $f(g(-1))$ and $g(f(-1))$.

(c) (4 pts.) Solve for x: $\dfrac{3}{2 - e^{-x/4}} = 5$.

(d) (4 pts.) If $\log_{10}(y+5) = (3x+1)\log_{10}(2)$, express y as a function of x. The answer should not contain logarithms or decimal approximations.

Problem 2 (14 points) You are preparing to bake a pizza at 230^oC. You hear a loud "POP" and notice the digital readout for the oven temperature has started to display changing temperatures. You record these temperatures over a 8 minute period. The resulting data is modeled by a function $P(t)$ computing the oven temperature (^{o}C) at time t minutes after the "POP". Assume these facts:

(i) Here is the graph of the function $P(t)$ on the domain $0 \leq t \leq 8$:



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(ii) The function $P(t)$ is sinusoidal during the time $0 \leq t \leq 8$ minutes.

(iii) The maximum oven temperature is 230^oC and first occurs at time $t = 0$. The minimum oven temperature is 200^oC and first occurs at time $t = 2$.

(a) (4 pts.) On the domain $0 \leq t \leq 8$, we know that the function has the form

$$P(t) = A \sin(\frac{2\pi}{B}(t - C)) + D,$$

for some constants A, B, C, D. Find these four constants. *In all four cases, you must explain how your got your numbers; no credit for answers only!*

Problem 2 (continued)

(b) (2 pts.) What is the oven temperature at time $t = 7.5$ minutes?

(c) (8 pts.) During the first 8 minutes, find all the times when the oven temperature is equal to 220^oC. You must clearly show all of your work. *(Note: Using a graphing calculator and "zooming" is not sufficient justification for your answer.)*

Problem 3 (14 points) On January 1, 1997, Mavis buys a small house in Yelm for \$59,000. On the same day in 1997, Clovis invests \$100,000 at his bank.

(a) (5 pts.) The house last sold on January 1, 1994, for \$42,000. Give an exponential model of the form $Y(t) = A_o b^t$ for the value of the house t years after Mavis buys it (i.e. in the year $1997 + t$).

(b) (2 pts.) What does the model predict Mavis could sell the house for after 4 years?

(c) (2 pts.) The bank pays Clovis 4.7%, compunded continuously. Give a function relating the value of his bank investment to time t (in years).

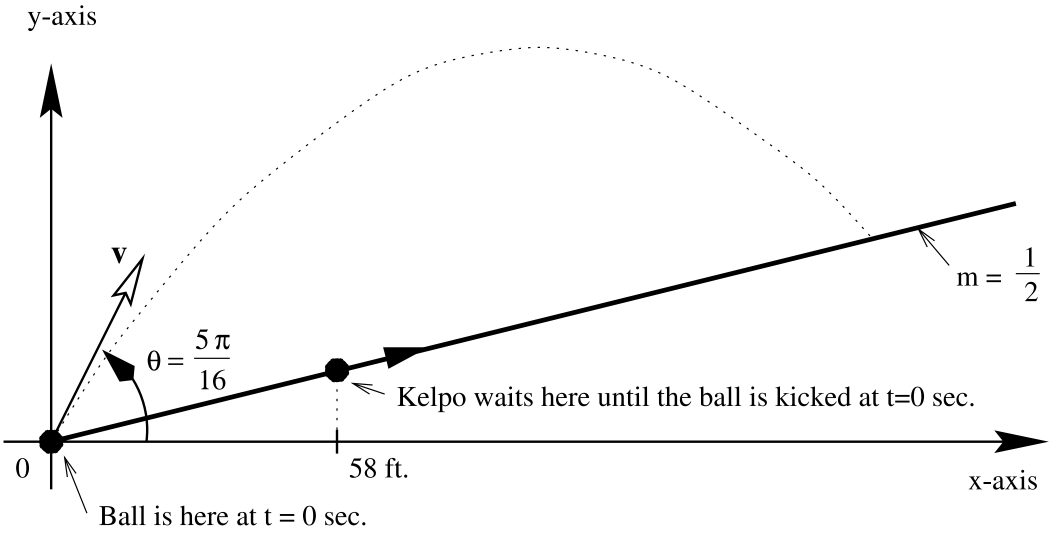
(d) (5 pts.) At what time will the value of Mavis' house equal the value of Clovis' investment?

Problem 4 (10 points) A vertical wheel has a 6 foot diameter and its center is 4 feet above the ground. A fly lands on the very top of the wheel. Then the wheel is spun clockwise at a constant rate of 20 RPM. Impose a coordinate system with the origin at ground level directly below the center of the wheel.

(a) (6 pts.) Give parametric equations describing the fly's position as a function of the elapsed time since it landed on the wheel (in seconds).

(b) (4 pts.) What are the fly's x and y coordinates after it has travelled 15 feet?

Problem 5 (18 points) Jamey is playing "fetch" with her dog, Kelpo. She is standing at the base of a hill that has a slope of $m = \frac{1}{2}$. Assume Jamey is standing at the origin of the coordinate system shown below. At $t = 0$ sec, Jamey kicks the ball on a trajectory toward Kelpo. She launches the ball in such a way that $|v| = 110\,\frac{\text{ft}}{\text{sec}}$ and $\theta = \frac{5\pi}{16}$ rad.



(a) (6 pts.) Write the parametric equations for the projectile motion of the ball using the coordinate system imposed in the picture.

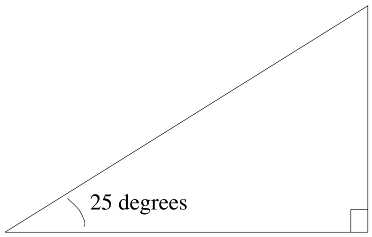
(b) (2 pts.) Write the equation, $y = f(x)$, describing the path of the ball.

Problem 5 continued

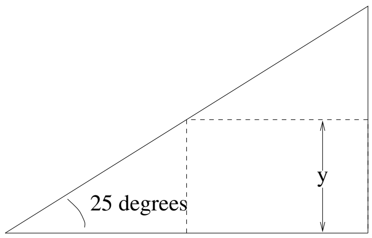
(c) (6 pts.) When and where does the ball land? *(Note: Using a graphing calculator and "zooming" is not sufficient justification for your answer.)*

(d) (4 pts.) Assume Kelpo is very fast and is able to run at a constant velocity up the hill. Kelpo starts running the instant the ball is kicked. How fast must Kelpo run to catch the ball at the instant the ball lands?

Problem 6 (10 points) A factory downtown which manufacturers flannel shirts sells their scrap material at bargain prices. The scraps are shaped like the right triangle below; the bottom edge is 30 inches long.

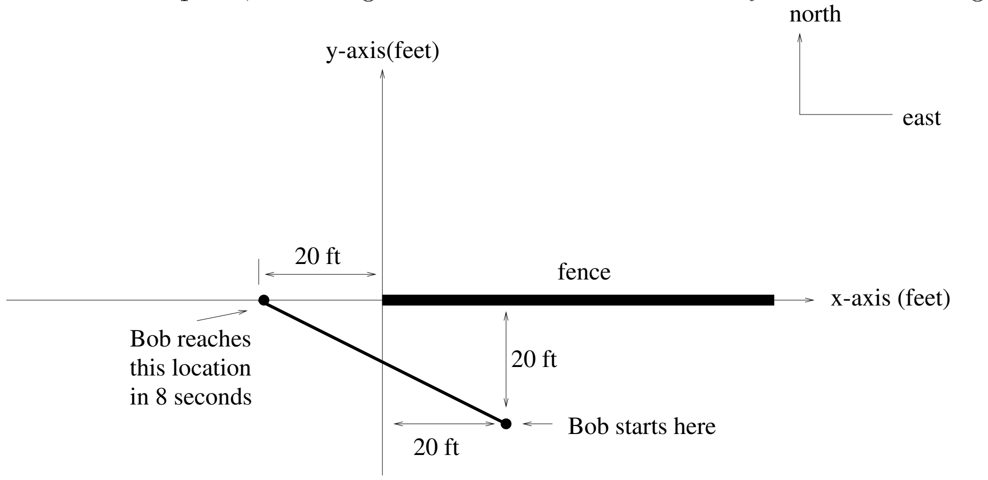


(a) (5 pts.) You buy some scraps to make a flannel quilt. You wish to cut out a rectangle from each fabric scrap that has one edge along the bottom of the fabric scrap, as pictured below, with y denoting the height of the rectangle. Find a formula which computes the area of the rectangular piece in terms of y.



(c) (5 pts.) What are the dimensions of the largest such rectangle you can cut from the fabric scrap? (That is, what are the dimensions of the rectangle which has the greatest area?)

Problem 7 (18 points) A fence with one end located at the origin in the xy-plane extends to the east as shown. Bob runs from the location 20 feet east and 20 feet south of the origin at constant speed, reaching the location 20 feet directly west of the origin 8 seconds later.



(a) (5 pts.) Find the coordinates of Bob after t seconds for $0 \leq t \leq 8$; i.e. find parametric equations for Bob's motion.

(b) (8 pts.) Find the distance from Bob to the closest point on the fence after t seconds. (*Hint: Think about which point on the fence is closest to Bob as he moves. The nature of the answer changes depending on whether Bob is east or west of the y axis, so the distance will be a multipart function.*)

Problem 7 (continued)

(c) (5 pts.) Find the time(s) when Bob is exactly 15 feet away from the fence.