a graphing calculator (or calculus) to minimize the length. Use common sense instead. [Hint: What is the shortest distance between two points?]

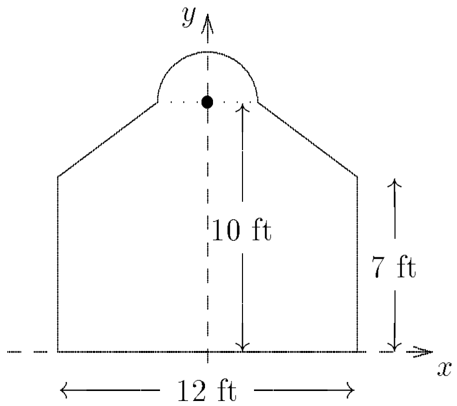
8. **[12 points total]** You have decided to write and sell a booklet called *The Math 120 Student Survival Guide*. A friend has done some marketing analysis for you. Her study shows that if you charge \$12, you will sell 500 copies. But, for each \$0.25 you lower the price, sales will increase by an additional 40 copies. (For example, if you charge \$11.75, you will sell 540 copies; if you charge \$11.50, you will sell 580 copies, etc.) Your startup cost to produce the booklet (things like a computer, printer, etc.) will be \$2800 and the additional production costs are \$3 **per book.** (Note: Your profit is computed by subtracting startup and production costs from the income you make on the total sales of the booklet.)

   (a) **[4 pts]** Find a formula for the number N of copies sold as a function of the sales price x.

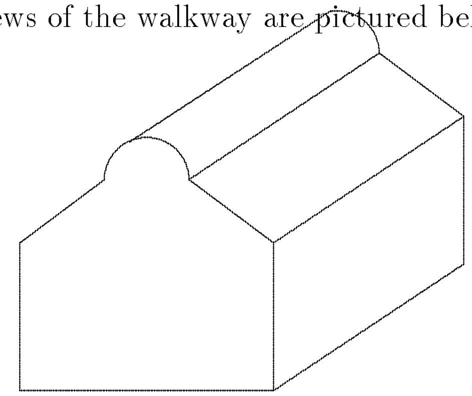
   (b) **[4 pts]** Find a formula for the profit P as a function of the sales price x.

   (c) **[4 pts]** What sales price for the booklet will maximize your profit? What will that profit be? *It is not sufficient to solve this by trial and error. You must use a mathematically valid method.*

9. **[12 points total]** A covered walkway is 12 feet wide. The roof over the walkway consists of a semicircular arch of diameter 4, centered 10 feet over the center of the walkway, together with flat pieces on either side that slope down to a height of seven feet at the left and right edges of the walkway. Two views of the walkway are pictured below.



End View

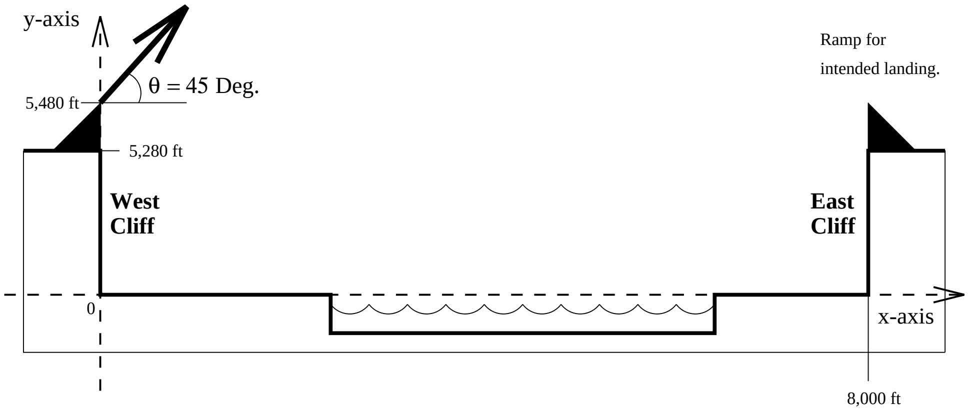


3–D View

   (a) **[8 pts]** Find the multipart formula for the height y of the roof as a function of the horizontal position x. Take the origin to be at the center of the walkway.

   (b) **[4 pts]** A man has two filing cabinets, one stacked on top of the other, on a cart. The total height of the cart and the two cabinets is 11 feet, and the cabinets are 3 feet wide. He wants to move the cabinets to an office at the other end of the walkway. Can he do so, or will he have to take off the top cabinet and come back for it? *Justify your answer!*

5. **[14 pts]** A vehicle is to be catapulted across Box Canyon. At $t = 0$ seconds, the vehicle launches off the ramp on top of West Cliff with $(x_0, y_0) = (0, 5480)$ and $v = 300$ mph. The launch direction is 45° above horizontal toward East Cliff. All distances are given in feet.



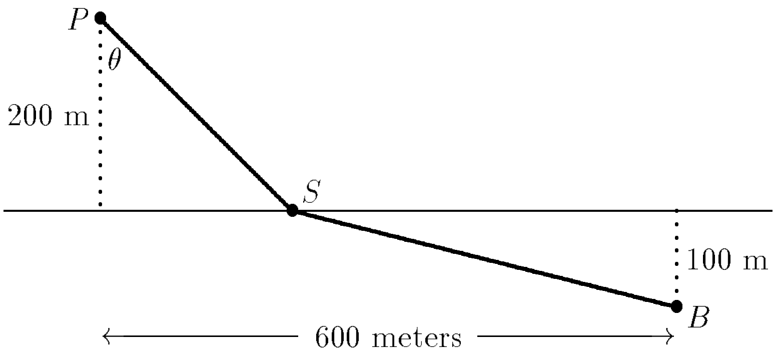
   (a) **[6 pts]** Write the parametric equations modeling the vehicle's motion.

   (b) **[4 pts]** What are the vehicle's x and y coordinates when it reaches its maximum height above the x-axis?

   (c) **[4 pts]** An on-board computer predicts a crash into East Cliff. If the flight is not stopped, when and where will the vehicle hit East Cliff? That is, give the time and xy–coordinates of the predicted crash point.

6. **[10 points total]** Particle A starts at the point (1,2) and moves in a straight line toward the point (-5,10) at a constant speed of 5 units/sec. Particle B starts at the point (0,2) and moves in a straight line at a constant speed toward the point (-4,0). It takes Particle B two seconds to move from (0,2) to (-4,0). Both particles start at the same time and move forever. When are they 7 units apart?

7. **[10 points total]** Suppose that you want to run a cable from a power source P out to a buoy B. The power source is located on land, 200 meters from a straight section of shoreline, and the buoy is located in the water, 100 meters out from a point 600 meters downshore from P. [See the picture.] The cable will run in a straight segment from P to a point S on the shoreline, and then from S in another straight segment out to B.



   (a) **[7 pts]** Find a formula for the **total length** l of cable as a function of the angle θ as shown in the picture.

   (b) **[3 pts]** What value of θ will minimize the total length? Note: you do not need to use

1. **[12 points total]** The price of a first class postage stamp in Fredonia grows according to an exponential model, $z = f(t)$, where z is the cost (in zlobeks) of a stamp and t represents the number of years since 1900. In 1955 a stamp cost 40 zlobeks and in 1980 it cost 112 zlobeks.
   (a) **[4 pts]** Find a formula for $f(t)$.
   (b) **[2 pts]** How much does a stamp cost this year?
   (c) **[4 pts]** Solve for t as a function of z. (i.e., find a formula for $t = f^{-1}(z)$, the inverse of the function you found in part (a).)
   (d) **[2 pts]** Describe in a complete sentence the **practical** meaning of the mathematical statement $f^{-1}(60) = 65$. Be sure to include units in your description.

2. **[10 points total]** A certain lake is stocked with fish. A standard model of population growth in an area of limited food resources leads to the following formula for the number N of fish in the lake t weeks after the fish are introduced to the lake:
$$N = \frac{750}{2 + 3e^{-(0.0896)t}}.$$
   (a) **[2 pts]** What is the number of fish initially?
   (b) **[6 pts]** After how many weeks has the number of fish doubled? *As always, show your work! You may not receive credit for answers obtained by graphical means or by using a "solver" function on your calculator.*
   (c) **[2 pts]** After a long time has passed, the population essentially stops growing and remains constant. What is the size of this steady-state population? (That is, what does N become when t becomes very large?)

3. **[12 points total]** For the last year, shares of stock in the Hartford Steam Boiler Company have been stuck in a trading range between \$37 and \$45 per share. Isobel wants to buy 100 shares, but for no more than \$40 per share. She has noticed that the stock regularly takes about 24 days to go from its low price to its high. It was selling at \$45 two days ago.
   (a) **[6 pts]** Create a sinusoidal model that gives the price per share of the stock $P(t)$ as a function of time t, in days. (Today is $t = 0$.)
   (b) **[2 pts]** What price does the model predict for 2 weeks from today?
   (c) **[4 pts]** During what time interval in the first cycle does the model predict the price will be below \$40 per share?

4. **[8 points total]** While Al is listening to his old 78 RPM Louis Armstrong records, his cat finds one of yesterday's macaroni noodles on the floor. She bats the noodle into the air and it lands on the spinning record, sticking at a point 5 inches West (i.e., to the left) of the center of the disk. The record is spinning clockwise at 78 revolutions per minute.
   (a) **[2 pts]** What is the linear speed (in inches per second) of the macaroni noodle, as it rotates with the record?
   (b) **[6 pts]** How long does it take for the noodle to travel a total of 245 inches? What are the x and y coordinates of the noodle at this time? Put the origin of your coordinate system at the center of the record.