letter

Math 120
Final, June 5
Spring 1995

Your Name ______________________

Your TA's Name ______________________

Instructions: Make sure your exam contains 10 pages; the cover page plus 9 exam pages. You may wish to initially scan the exam to plan your strategy. All work must be on this exam. Good luck.

Scoring:

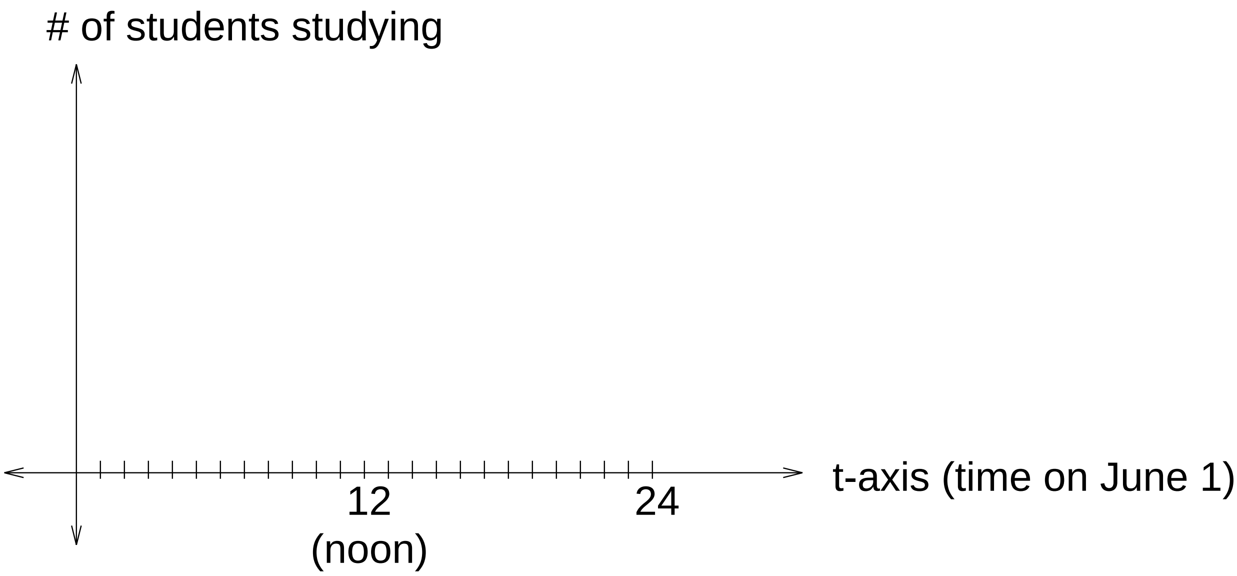
PROBLEM	POINTS SCORED
1	/ 18
2	/ 22
3	/ 20
4	/ 24
5	/ 16
TOTAL	/ 100

Problem #1 (18 pts.) Aaron has a mathematical model which predicts the number of Math 120 students studying at time t on June 1; t= hours past midnight. The maximum number of students studying during the day was 50, first occurring at 10 am. The minimum number of students studying during the day was 2, first occurring at 4am. Aaron's model is a sinusoidal function

$$a(t) = A\sin(\frac{2\pi}{B}(t-C)) + D,$$

for constants A, B, C, D.

(a)(2 pts.) Plot the two relevant data points on the graph which are given in the problem:



(b)(2 pts.) Find A and D.

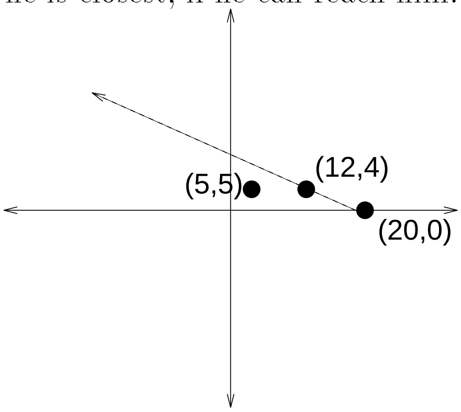
Problem #1 (cont.)

(c)(6 pts.) Find B and C.

(d)(2 pts.) How many Math 120 students are studying at 2pm ?

(e)(6 pts.) At what time(s) during June 1 will exactly 44 students be studying? (Hint: There may be more than one answer.)

Problem #2 (22 pts.) Rob Dabank is running away from his latest crime. He exits the Bank at the point (20 meters, 0 meters) in an xy-coordinate system. He runs at a constant speed in a straight line. After 2 seconds he is at the point (12 meters, 4 meters). Lay Z Copper, the police officer, is standing at (5,5). All he has is a 2 meter stick to trip Rob Dabank. Lay Z Copper will trip Rob Dabank when he is closest, if he can reach him...



(a)(6 pts.) Find parametric equations for the motion of Rob Dabank. Where is Rob Dabank located after 0.6 seconds?

(b)(6 pts.) Find a formula for the distance SQUARED $d^2(t)$ from Lay Z Copper to Rob Dabank as a function of time t.

Problem #2 (cont.)

(c)(6 pts.) When is the distance squared, $d^2(t)$, a minimum?

(d)(4 pts.) Will Lay Z Copper trip Rob Dabank and thus save the day?

Problem #3 (20 pts.) Scientists use radiocarbon dating to determine when a living object died. When an object dies, the amount of radioactive carbon-14 present decays as a function of time. By determining how much of an object's carbon-14 is left, its date of death can be measured. If the amount of carbon-14 present at death is P, the amount which remains x years after death is given by

$$y = Pe^{-.00012378x}.$$

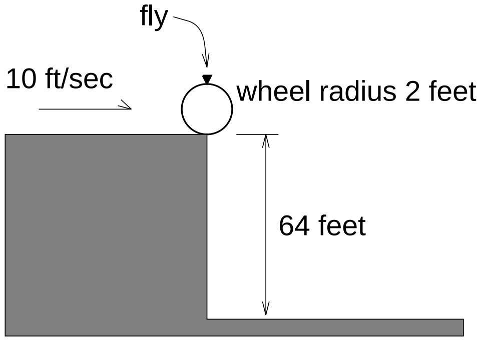
(a)(5 pts.) An antique dealer in New York claims to have a piece of scratch paper used by Columbus during his voyage to America in 1492. The paper is made of cotton fibres. If the cotton which was used died 500 years ago, what percentage of the carbon-14 would be left?

(b)(5 pts.) After testing the paper, the fraction of carbon-14 left is 97% of the amount originally present. Determine when the cotton used to make the paper died.

Problem #3 (cont.) (c)(5 pts.) The *half life* of a radioactive substance is the amount of time it takes for the amount of the substance to be cut in half. Determine the half-life of carbon-14.

(d)(5 pts.) The half-life of the element Michaelonium is 4 hours. Find a constant a so that the function $f(x) = e^{ax}$ gives the fraction of Michaelonium present after x hours.

Problem #4 (24 pts.) In order to avenge all the nasty problems involving combined parametrized motion and the fly, Math 120 students stick a piece of fly paper on a 2 ft. radius wheel travelling 10 ft/sec. The fly lands on top of the wheel (getting stuck to the fly paper) just as it's about to roll off a cliff that's 64 ft high. Place the origin at the bottom of the cliff wall.



(a)(4 pts.) Find the parametric equations of the center of the wheel $x_p(t), y_p(t)$. (Caution: Note the y-value of the wheel center.)

(b)(3 pts.) When does the wheel hit the ground? (Caution: Note the y-value of the wheel center when the wheel hits the ground.)

(c)(2 pts.) Find the angular speed ω of the fly.

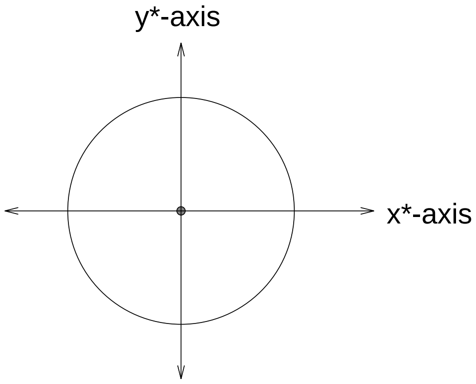
Problem #4 (cont.)

(d)(4 pts.) Find the parametric equations of the fly relative to the center of the wheel $x^*(t), y^*(t)$.

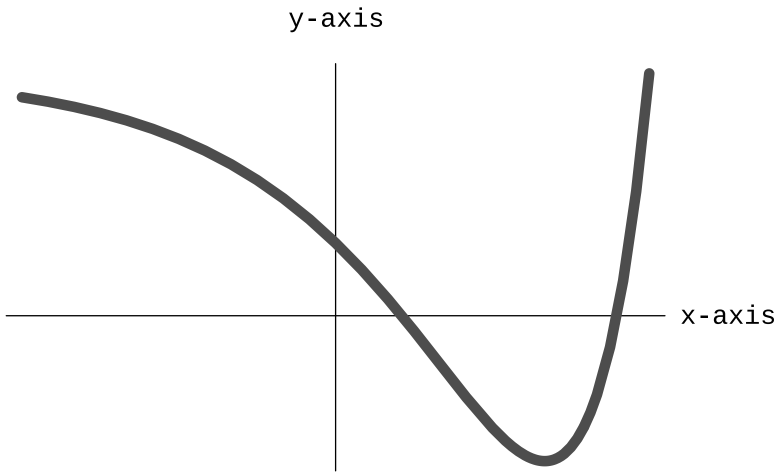
(e)(4 pts.) Give parametic equations for the fly.

(f)(3 pts.) What is the position (x, y coordinates) of the fly when the wheel hits the ground?

(g)(4 pts.) Consider the moving x^*y^*-coordinate system with origin at the wheel center. In the picture below, indicate the x^*y^*-coordinates of the fly on the wheel when the wheel hits the ground AND find the measure of the angle which corresponds to this position.



Problem #5 (16 pts.) Below is a picture of a portion of the graph of $y = f(x) = 2^{2x} - 8(2^x) + 10$.



(a)(2 pts.) What are the xy-coordinates of the point where the graph crosses the y-axis?

(b)(8 pts.) What are the xy-coordinates of the two points where the graph crosses the x-axis? (Hint: You will need to replace "2^x" by "z" and solve a quadratic.)

(c)(6 pts.) Find the xy-coordinates of the lowest point on the graph.