Your Name ____________________

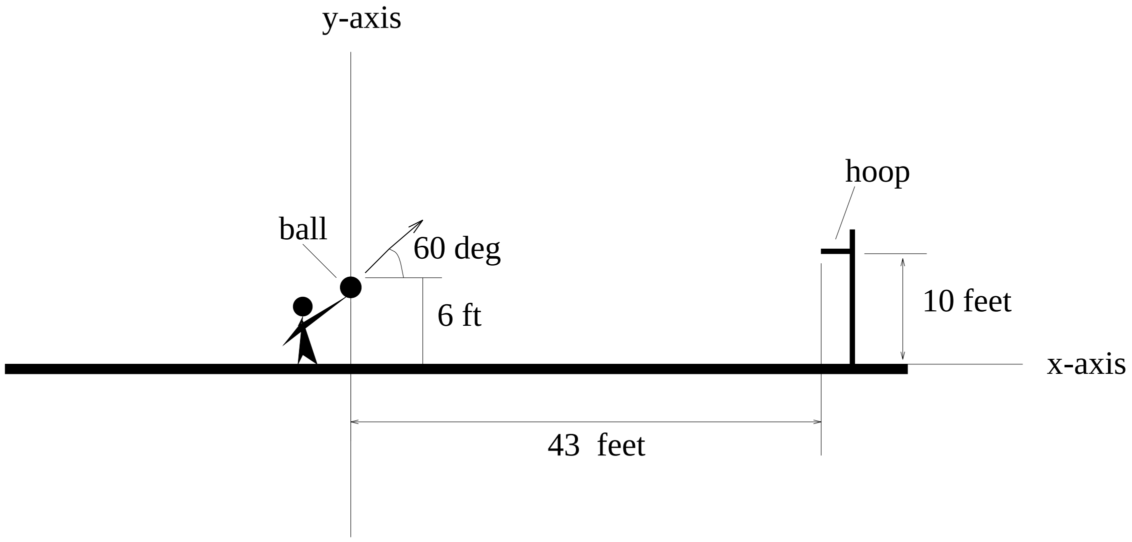
Your TA Section ____________________

Math 120
Final, December 9, 1995
Autumn 1995

!!! READ-INSTRUCTIONS-READ !!!

1. The exam contains 8 problems. The problems are NOT of equal point value. You can see the point value of each problem tabulated below. A perfect score is 120. The point value for each part of a problem is clearly indicated.
2. Make sure your exam contains 13 pages; the cover page plus 12 exam pages.
3. You may wish to initially scan the exam to plan your strategy.
4. All work must be on this exam. NO CREDIT for answers only.
5. You have 3 hours. If you have a question, please raise your hand and someone will assist you as soon as possible.

Problem #1:(22 pts.) Just as a basketball game is ending, Lynn shoots the ball from the position indicated below. The ball is launched with an initial speed of 40 ft/sec at an angle 60^o above horizontal. Lynn is 43 ft. away from the front of the rim of the hoop; the hoop is 10 ft. high. Impose coordinates as pictured.



(a) (4 pts.) Find the horizontal velocity v_x and the vertical velocity v_y of the ball when it is launched.

(b) (4 pts.) Let $P(t) = (x(t), y(t))$ be the coordinates of the ball t seconds after it is launched. Find the functions $x(t)$ and $y(t)$.

(c) (6 pts.) Find where (xy-coordinates) the ball reaches its highest point.

(d) (6 pts.) Find all location(s) where the ball is 10 feet above the floor; give the xy-coordinates.

(e) (2 pts.) Does the ball go through the hoop (you must give a reason)?

Problem #2: (12 points) The length of some fish are modeled by a *von Bertalanffy* growth function. For Pacific halibut, this function has the form

$$L(t) = 200\,(1 - 0.956\,e^{-0.18t})$$

where $L(t)$ is the length (in centimeters) of a fish t years old.

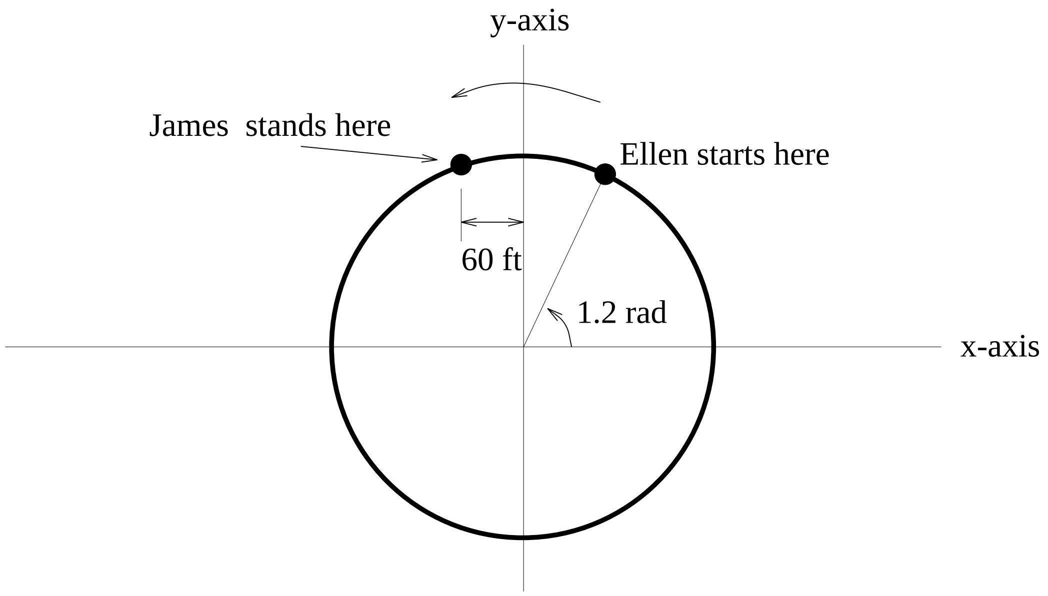
(a) (3 pts.) What is the length of a new-born halibut at birth?

(b) (2 pts.) Use the formula to estimate the length of a 6–year–old halibut.

(c) (4 pts.) At what age would you expect the halibut to be 120 cm long?

(d) (3 pts.) What is the practical (physical) significance of the number 200 in the formula for $L(t)$?

Problem #3:(22 points) Ellen begins running counterclockwise around a circular track of radius 200 ft. Her starting location is as pictured. Ellen's angular speed is $\omega = 0.0757$ rad/sec. Impose a coordinate system whose origin is the center of the circular track. James is standing at the indicated spot.



(a) (2 pts.) How long does it take Ellen to complete one lap; i.e. once around the track, back to her starting location?

(b) (2 pts.) How fast (in feet/sec) is Ellen running?

(c) (6 pts.) When does Ellen first cross the x-axis **AND** how far has she run when this happens?

(d) (6 pts.) Let $P(t) = (x(t), y(t))$ be the location of Ellen at time t; find the formulas for $x(t)$ and $y(t)$.

(e) (2 pts.)Where (xy-coordinates) is Ellen located after 38 seconds?

(f) (4 pts.) When does Ellen first pass James?

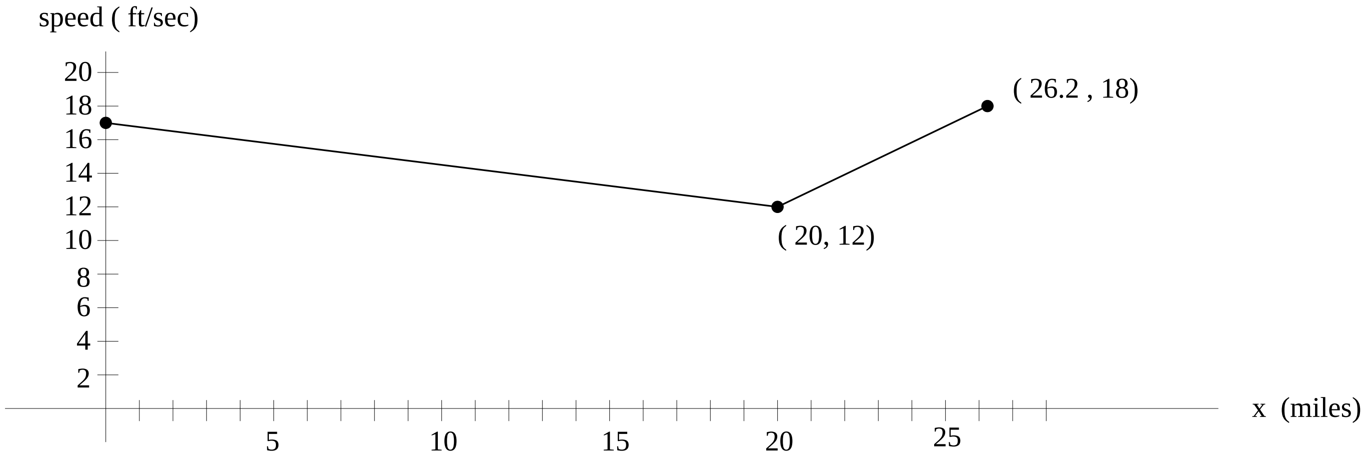
Problem #4: (15 points) Some population models (especially in predator–prey situations) lead to populations which vary sinusoidally. Suppose you begin observing the population of a certain aquatic predator and make the following estimates: The maximum population was 14,000 and it occured at $t = 6$ months. Fourteen months later (at $t = 20$ months), the population bottomed out at a minimum of 6,000. Assume that the population $P(t)$ varies sinusoidally with time.

(a) (3 pts.) Sketch a graph of P versus t over the interval $0 \le t \le 36$ months.
Label the coordinates at each maximum and minimum of $P(t)$.

(b) (6 pts.) Find a formula for $P(t)$, the population after t months.

(c) (6 pts.) When is the first time after $t = 6$ that the population was 13,000?

Problem #5:(14 pts.) Marathon runners like to keep a careful listing of their performance during the 26.2 mile race. Here is a plot of Tim's speed (in units of "feet/sec") at mile x during a marathon. He starts the race running 17 ft/sec. This graph consists of two line segments between the indicated points.



Let $s(x)$ be the function which tells us the runners speed at mile x in the race.

(a) (2 pts.) What is a formula for $s(x)$ during the first 20 miles of the race?

(b) (2 pts.) What is a formula for $s(x)$ during the last 6.2 miles of the race?

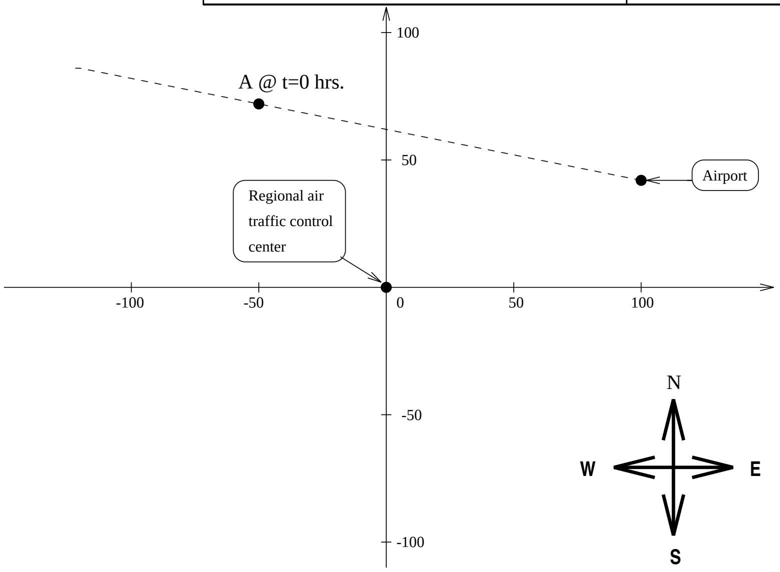
(c) (2 pts.) What is Tim's speed at mile 12?

(d) (6 pts.) During what portion(s) of the race is Tim's speed greater than 15 feet/sec?

(e) (2 pts.) In reality, runners tend to think in terms of "pace", which is defined to be the number of minutes required to run one mile. (For example, 6 min/mile is a very respectable marathon pace; Olympic atheletes can maintain a 5 min/mile pace.) What is Tim's pace in units of "min/mile" at mile 12?

Problem #6: (17 pts.) The Regional Air Traffic Control Center (at the origin) has the following partial set of data on two aircraft approaching a local airport. The airport is 100 miles East and 40 miles North of the control center. *Note: All distances are in "miles", and all speeds are in "miles per hour".*

Known data from radar:	Aircraft A	Aircraft B
Initial coordinates $t = 0\,hrs$	$(-50\,mi, 70\,mi)$	To be determined.
Speed	$100\,mph$	To be determined.



(a) (6 pts.) What are the parametric equations for aircraft A? (Use 2 decimal places of accuracy.)

(b) (5 pts.) At $t = 0$, the pilot transmits sufficient information to derive the following parametric equations for aircraft B: $x_B(t) = -100 + 96t$ and $y_B(t) = -100 + 67.2t$.

- What are the coordinates of aircraft B when the pilot transmits its position?
- What is the speed of aircraft B?

(c) (4 pts.)

- When does plane A land? Write your answer to at least 4 decimal places.
- When does plane B land? Write your answer to at least 4 decimal places.

(d) (2 pts.) Approximately where is the second aircraft the instant the first aircraft lands. Write the coordinates in decimal miles at least to 2 decimal places.

Problem #7: (10 pts.) A sailing club charges its members \$100 per year in dues. The fee **for every member** is reduced by \$1 for each member in excess of 60. Thus, for example, if the club had 65 members, the fee for each of those 65 members would be reduced by \$5 ($= \$1 \times (65 - 60)$), and so each member would pay \$100-\$5=\$95 per year.

(a) (6 pts.) If x represents the number of members in the club, find a formula for the total dues revenue which is valid when $x > 60$.

(b) (4 pts.) What number of members would maximize the dues revenue?

Problem #8: (8 pts.) In 1968, the U.S. minimum wage was \$1.60 per hour. In 1976, the minimum wage was \$2.30. Assume that the minimum wage is growing according to an exponential model $w(t)$, where t represents time in years since 1960.

(a) (4 pts.) Find a formula for $w(t)$?

(b) (2 pts.) Congress is currently debating legislation to raise the minimum wage to \$5.15 in 1996. Is this legislation above, below or equal to what the model predicts?

(c) (2 pts.) According to the model, during what year would the minimum wage first reach \$10 per hour?