Midterm 1 Key

Monday, October 21, 2024 2:47 PM

Math 120 Section **Midterm I, October 24th** **Fall 2024**

HONOR STATEMENT
I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam.

Name

Signature

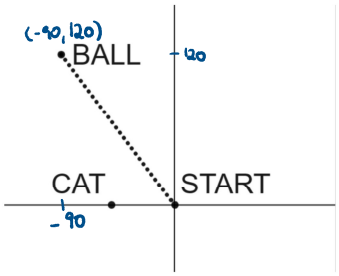
Student ID #

- Silence your phone and put it away.
- You have 50 minutes for 4 problems. Check your copy of the exam for completeness.
- You are allowed to use a hand written sheet of paper (8x11 in), back and front.
- Calculator : TI 30 XIIS.
- Justify all your answers and show your work for credit.
- All answers must be exact, no rounding.
- Each problem is worth 15 points.

Do not open the test until everyone has a copy and the start of the test is announced.

GOOD LUCK!

Problem 1. A dog is running on a straight line with constant speed to retrieve a ball. The ball is located 90 meters west and 120 meters north of the dog's starting point. The dog starts running at $t = 0$ and reaches the ball 15 seconds later. Do not round in this problem and put a box around your final answers.



(a) Find the parametric equations of the dog's uniform linear motion.

$x = at + b$

$y = ct + d$

@ $t=0$, $(0,0)$ $\rightarrow$ $0 = b$, $0 = d$

$t = 15$, $(-90, 120)$ $\rightarrow$ $-90 = a \cdot 15$, $120 = b \cdot 15$

$\Rightarrow$ $a = -6$, $b = 8$

$$\boxed{\begin{aligned} x &= -6t \\ y &= 8t \end{aligned}}$$

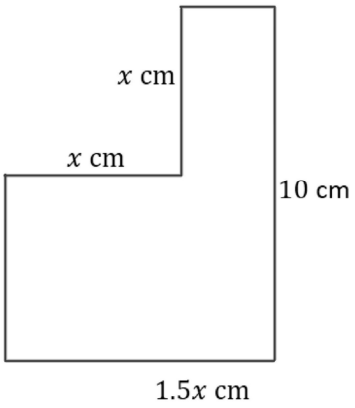
(b) When **and** where (two answers!) will the dog be due north of a cat who lies 50 m west of the dog's starting point?

$x = -50$: $-50 = -6 \cdot t \Rightarrow t = \frac{50}{6}$ seconds

$y = 8t \rightarrow y = 8 \cdot \frac{50}{6} = \frac{200}{3}$

at $t = \frac{50}{6}$ seconds, the dog's coordinates are $(-50, \frac{200}{3})$

Problem 2. From a rectangle of $1.5x$ cm width and 10 cm length a square with side length x has been cut out, as illustrated in the sketch.



(a) Find a formula for the area of the shape in terms of x. Put a box around your final answer.

$A = 1.5x \cdot 10 - x^2$

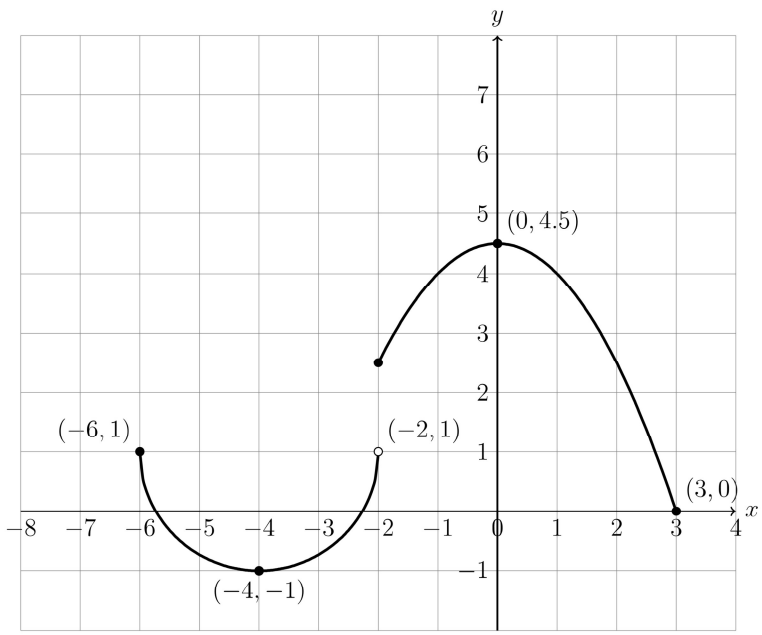
$\boxed{A = -x^2 + 15x}$

(b) How do you have to choose x to maximize the area? Put a box around your final answer.

The area is a prabola, opened down, with the maximum @ the vertex:

Vertex @ $x = -\frac{15}{-2} = 7.5$

Problem 3. (a) Below is the graph of a piecewise (multipart) function $f(x)$ and five given points. Its left piece is a semicircle its right piece is a quadratic function passing through vertex $(0, 4.5)$ and the point $(3, 0)$. Find the piecewise (multipart) formula for $f(x)$.



$y = a(x)^2 + 4.5$

$0 = 9a + 4.5$

$9a = -4.5$

$a = -\frac{1}{2}$

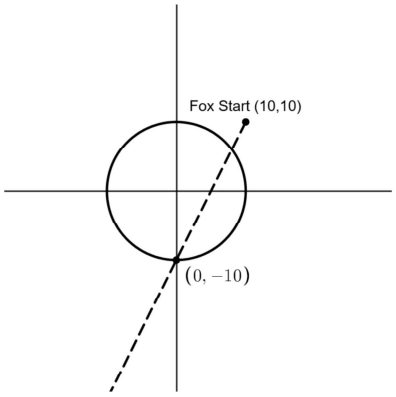
$$f(x) = \begin{cases} 1 - \sqrt{4 - (x+4)^2} & \text{if } -6 \le x < -2 \\ -\frac{1}{2}(x)^2 + 4.5 & \text{if } -2 \le x \le 3 \end{cases}$$

(b) Find the domain **and** range of $f(x)$ from part (a). Put a box around your final answer.

domain: $-6 \leq x \leq 3$ or $[-6, 3]$

range: $-1 \leq y \leq 4.5$ or $[-1, 4.5]$

Problem 4. A motion sensor can detect movement in a circular region of radius 10 meters. A fox trots on a straight line from a point 10 meters north and 10 meters west of the sensor towards the most southern point of the sensor (see sketch).



(a) Where will the fox enter the detection zone? Put a box around your final answer.

circle: $x^2+y^2=100$

fox: slope: $(10,10)\to(0,-10)$ $m=\frac{-10-10}{0-10}=2$

$y=2x-10$

intersect: $x^2+(2x-10)^2=100$

$x^2+4x^2-40x+100=100$

$5x^2-40x=0 \quad |\div 5$

$x(x-8)=0$

$x=0$ or $x=8$ ∴ We look for $x=8$.

$\Rightarrow y=2\cdot 8-10=6$

$\boxed{(8,6)}$

(b) Assume that the fox's speed is constant at 1.5 meters per second. To save battery, the sensor turns on for 10 seconds then stops for 10 seconds then turns back on for 10 seconds, etc. If it turns off the moment the fox enters the zone, will the fox be detected by the sensor? Show all your work.

need to find the time the fox is within zone:

$(8,6) \to (0,-10)$ distance:

$$d = \sqrt{(8-0)^2 + (6+10)^2} = \sqrt{64 + 16^2}$$

$$= \sqrt{320} = 8\cdot\sqrt{5} \sim 17.88 \text{ meters}$$

time in zone: $v = \frac{d}{t}$ $t = \frac{d}{v} = \frac{17.88 \text{ meters}}{1.5 \text{ meters/s}}$

~ 11.93s

$\Rightarrow$ As the fox is >10s in circle, it will be detected

Scratchpaper