Math 120 Sections A/B **Midterm II, November 14th** **Winter 2024**
Instructor: Natalie Naehrig

HONOR STATEMENT
I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam.

Name

Signature

Student ID #

- Silence your phone and put it away.
- You have 50 minutes for 4 problems. Check your copy of the exam for completeness.
- You are allowed to use a hand written sheet of paper (8x11 in), back and front.
- Calculator : TI 30 XIIS.
- Justify all your answers and show your work for credit.
- Unless otherwise stated, all answers must be exact, no rounding.
- Each problem is worth 15 points.
- The last page is for scratch paperwork and will not be graded unless you indicate so.

Do not open the test until everyone has a copy and the start of the test is announced.

GOOD LUCK!

Problem 1. Consider the function $f(x) = \frac{1}{2}(x+2)^2 - 1$ with domain $-6 \leq x \leq -2$. The graph of f is shown on the next page.

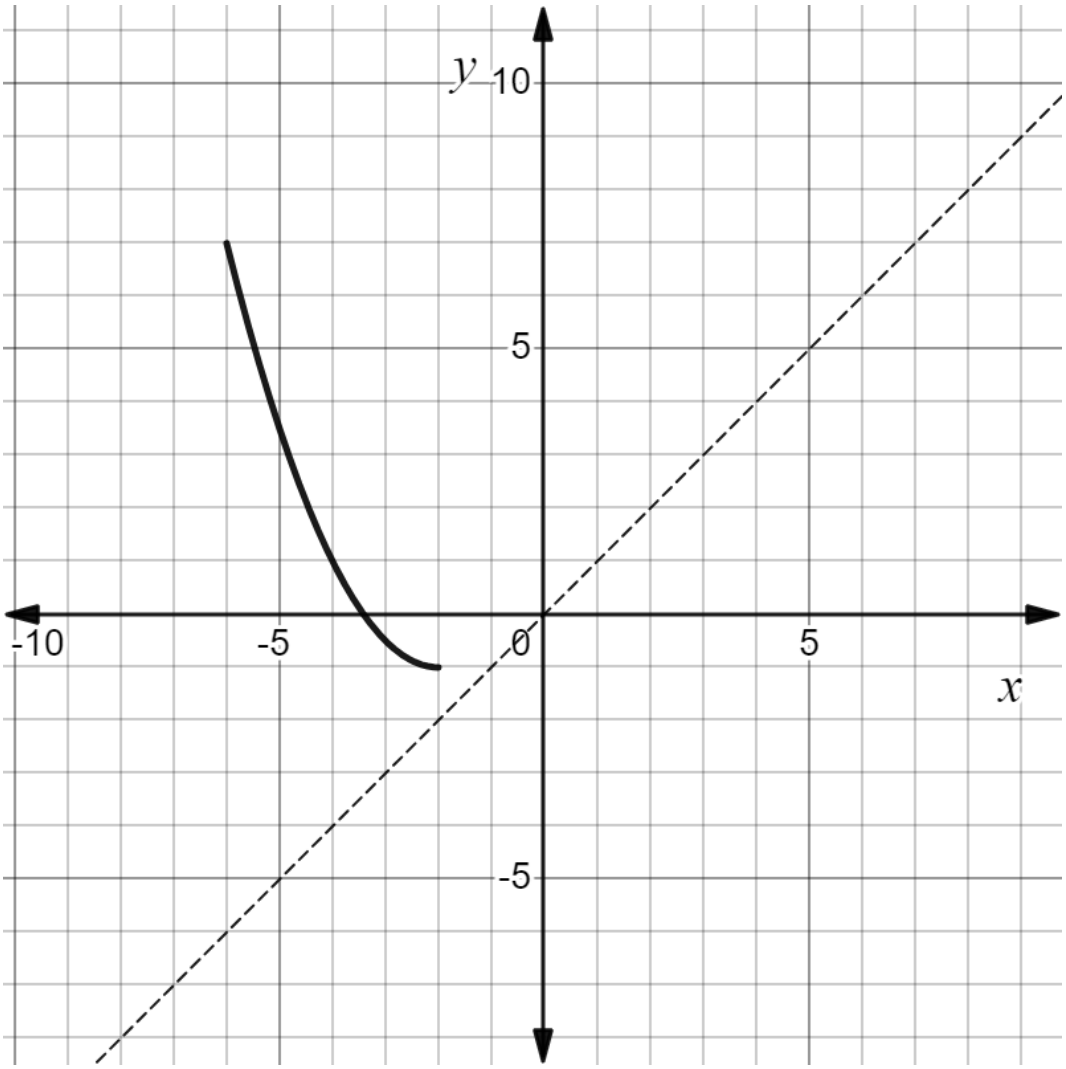
(a) Compute the range of $f(x)$. Put a box around your final answers.

(b) Justify why $f(x)$ is invertible. You may use the graph of $f(x)$ on page 3 to reason.

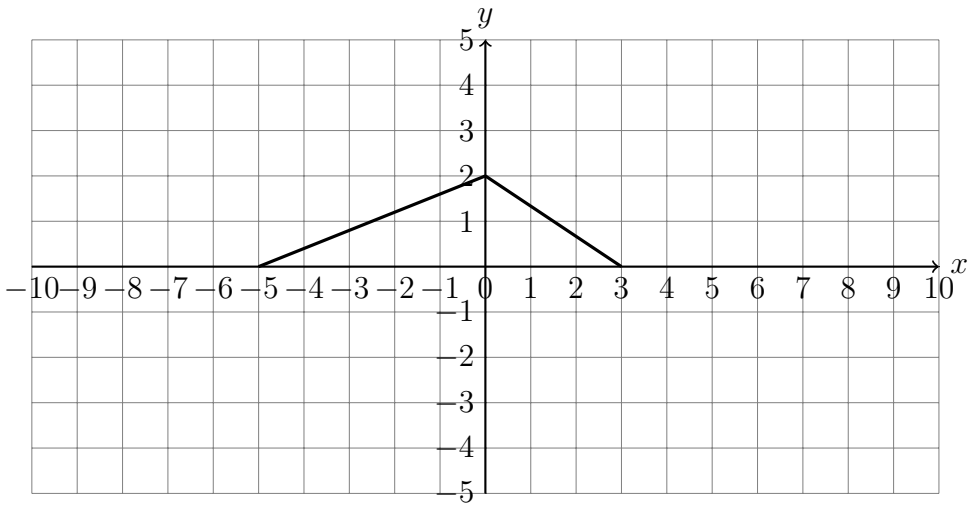
(c) What are domain and range of $f^{-1}(x)$? Put a box around your final answers.

(d) Find the inverse function $f^{-1}(x)$ of $f(x)$. Put a box around your final answers.

(e) Using the diagonal $y = x$, sketch the inverse function of $f(x)$ in the given coordinate system. At least 3 points of the graph should be precise and labeled.



Problem 2. Consider a function $f(x)$ whose graph is shown below.



(a) What are the domain **and** range of $f(x)$? Put a box around your final answers.

(b) Identify the transformations f underwent to obtain $g(x) = -2f(2x - 1)$. To do so, mark all circles that apply, leave those blank that do not apply. Write numbers 1, 2, 3, ... in front of the relevant transformations to indicate the order they were applied.

.....○ Horizontal translation by ..... units to the ○ left ○ right.

.....○ Vertical translation by ..... units to the ○ up ○ down.

.....○ Horizontal dilation by factor ..... (It is a ○ compression ○ expansion).

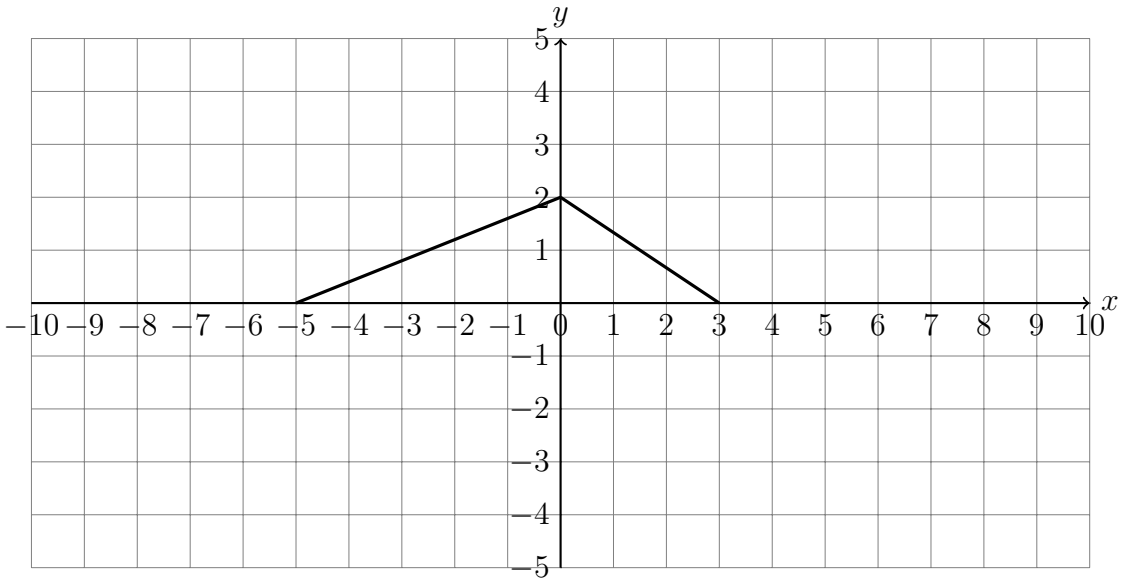
.....○ Vertical dilation by factor ..... (It is a ○ compression ○ expansion).

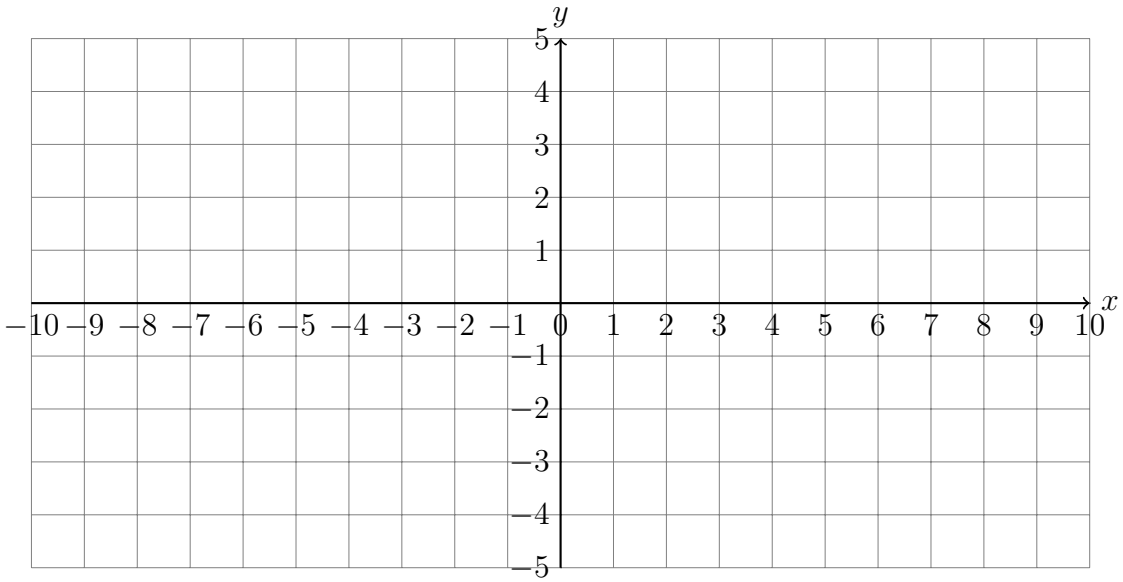
.....○ Reflection about the x-axis.

.....○ Reflection about the y-axis.

(c) Find the domain of $g(x)$ through algebra (use this as a confirmation with your graph on the next page). Put a box around your final answers.

(d) The original graph of $f(x)$ as well as a blank coordinate system have been provided. Use the first coordinate system to perform one transformation after the other. Use the blank coordinate system to present your final graph of $g(x) = -2f(2x - 1)$.





Problem 3. The population growth of City A and City B follows an exponential model. At the beginning ($t = 0$, t in years), City A had a population of 24,000 people with a doubling time of 50 years. City B has an initial population of of 50,000 people and grows by 5% in 5 years.

(a) Set up the exponential functions that model City A's and City B's population. The variable t should be in years. Round to 6 decimal places. Put a box around your final answers.

(b) When will the population of City A be equal to that of City B? Round to the nearest year. Put a box around your final answer.

(c) What is the doubling time of City B's population? Round to the nearest year. Put a box around your final answer.

Problem 4. Consider the functions $f(x) = \ln(x-1)$ and $g(x) = \sqrt{2x-4}$.

(a) What are the domains of $f(x)$ and $g(x)$. Put a box around your final answers.

(b) Find $f(g(x))$ and $g(f(x))$. Put a box around your final answers.

(c) Find the domains of $f(g(x))$ and $g(f(x))$. Put a box around your final answers.