

Fall 2022 Math 120 A midterm 1

NAME (First,Last) :

Student ID

- Please use the same name that appears in Canvas.
- IMPORTANT: Your exam will be scanned: DO NOT write within 1 cm of the edge. Make sure your writing is clear and dark enough.
- IMPORTANT : remove the last page (Scratch paper) before turning in the exam.
- IMPORTANT: Write your NAME (first, last) on top of the third page of this exam.
- Unless stated otherwise, you **MUST** show work for credit.
- Your work needs to be neat and legible.
- Unless the problem gives you different instructions, you can give exact answers or round off your answers to 2 decimal places.
- The only calculator allowed is the TI 30X IIS. You are allowed an 8x11 sheet of notes, written both sides.
- Box your final answer, when appropriate.
- Raise your hand if you have a question.

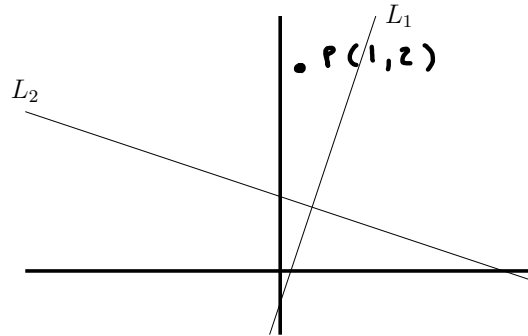
1. The pictures in this problem are not to scale. Do not try to estimate anything graphically. Use formulas and show your work.

(a) The lines below are perpendicular and intersect at $P(1, 2)$. The equation of L_1 is $y = 2 + 3(x - 1)$. What is the equation of L_2 ?

$$m = -\frac{1}{3}$$

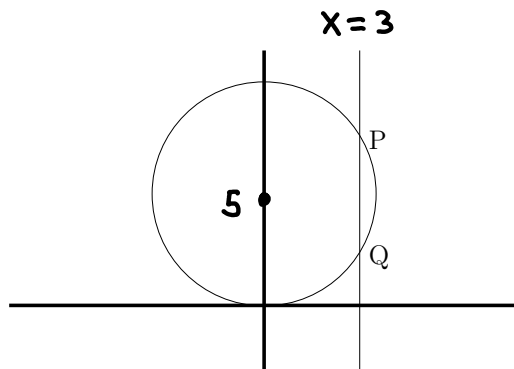
$$y = 2 - \frac{1}{3}(x - 1)$$

$$\text{or } y = -\frac{1}{3}x + \frac{7}{3}$$



or correct equation in any format

(b) The circle below has radius $r = 5$. The origin of the coordinate system is the lowest point of the circle. The line $x = 3$ intersects the circle at P and at Q .



(i) What is the equation of the circle?

$$x^2 + (y - 5)^2 = 5^2$$

(ii) What is the y coordinate of Q ?

plug in $x = 3$ and solve for y :

$$3^2 + (y - 5)^2 = 5^2$$

$$(y - 5)^2 = 16$$

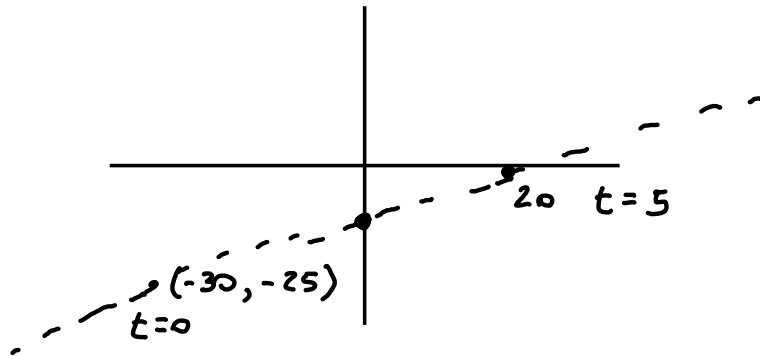
$$y = 5 \pm 4$$

$$y = 1$$

NAME (First Last):

2. A rabbit's burrow is located 30 mi to the West and 25 mi to the South of Ann's house. At $t=0$, the rabbit leaves his burrow and runs in a straight line, with constant speed, towards a tree located 20 mi East of the house, reaching the tree 5 hours later.

- (a) Introduce a coordinate system with the origin at Ann's house, and draw a picture of the rabbit's path. Show the coordinates of the rabbit's burrow, Ann's house and of the tree.



- (b) Find the parametric equations of motion for the rabbit.

$$x(t) = -30 + \frac{20 - (-30)}{5 - 0} t = -30 + 10t$$

$$y(t) = -25 + \frac{0 - (-25)}{5 - 0} t = -25 + 5t$$

- (c) Assume the rabbit stops running when he reaches the tree and sits there (forever). For what time interval are the parametric equations you found in part b) valid, that is, give the correct position of the rabbit?

$$0 \leq t \leq 5 \quad (\text{This is when the rabbit is moving})$$

PROBLEM 2 CONTINUED

- (d) From time $t = 0$ to time $t = 3$ Ann is at home. 3 hours after the rabbit starts running, Ann leaves her house heading North at a speed of 6 mph. Ann stops walking and stands still (forever) at time $t=5$. Find all times when the distance between Ann and the rabbit is 12 mi. Give your answer in decimal notation, rounding off to 2 digits.

$$\text{Rabbit } (-30 + 10t, -25 + 5t) \\ \text{for } 0 \leq t \leq 5$$

$$\text{Ann } (0, 6(t-3)) \\ \text{for } 3 \leq t \leq 5$$

$$\sqrt{(-30 + 10t)^2 + (-25 + 5t - 6(t-3))^2} = 12, \text{ if } 3 \leq t \leq 5$$

$$900 - 600t + 100t^2 + (-25 - t + 18)^2 = 144$$

$$900 - 600t + 100t^2 + t^2 + 14t + 49 - 144 = 0$$

$$101t^2 - 586t + 805 = 0$$

$$t = \frac{586 \pm \sqrt{586^2 - 4 \cdot 101 \cdot 805}}{202} = \cancel{2.23}, \boxed{3.57}$$

For $0 \leq t \leq 3$ Ann stays at $(0, 0)$:

$$\sqrt{(-30 + 10t)^2 + (-25 + 5t)^2} = 12 \quad 0 \leq t \leq 3$$

$$900 - 600t + 100t^2 + 625 - 250t + 25t^2 - 144 = 0$$

$$125t^2 - 850t + 1381 = 0$$

$$t = \frac{850 \pm \sqrt{850^2 - 4 \cdot 125 \cdot 1381}}{250} = \boxed{2.68} \quad \cancel{4.11}$$

SCRATCH PAPER : WORK DONE HERE WILL NOT BE GRADED. PLEASE REMOVE THIS PAGE BEFORE TURNING IN THE EXAM.