Fall 2022 Math 120 A midterm 2

NAME (First,Last) : .....

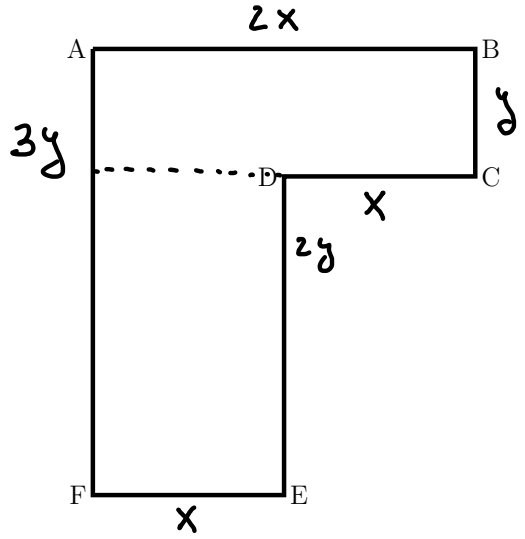
UW email: .....

Student ID .....

Section .....

- Please use the same name that appears in Canvas.
- IMPORTANT: Your exam will be scanned: DO NOT write within 1 cm of the edge. Make sure your writing is clear and dark enough.
- IMPORTANT : do not turn in any scratch paper.
- IMPORTANT: Write your NAME (first, last) on top of the third page of this exam.
- If you run out of space, continue your work on the back of the second page and indicate clearly on the problem page that you have done so.
- Unless stated otherwise, you **MUST** show work for credit.
- Your work needs to be neat and legible.
- Unless the problem gives you different instructions, you can give exact answers or round off your answers to 2 decimal places.
- The only calculator allowed is the TI 30X IIS. You are allowed an 8x11 sheet of notes, written both sides.
- Box your final answer, when appropriate.
- Raise your hand if you have a question.

1. You would like to build a rectangular enclosure which has an L shape as shown below and a total perimeter of 60 feet.



The length of AB is double the length of DC, and the length of AF is triple the length of of BC. (the picture above is not to scale, so do not use it to estimate anything graphically). What is the maximum possible area of the enclosure? Choose variables and mark clearly the lenght of each side of the enclosure. (hint: consider the enclosure as made up by two rectangles)

$A = 2x \cdot y + x \cdot 2y = 4xy$

$60 = 2x + y + x + 2y + x + 3y = 4x + 6y$

$\frac{60 - 4x}{6} = y$

$A = 4x\left(10 - \frac{2}{3}x\right) = 40x - \frac{8}{3}x^2$  Max at vertex

$h = \frac{40}{\frac{16}{3}} = \frac{120}{16} = \frac{15}{2}$

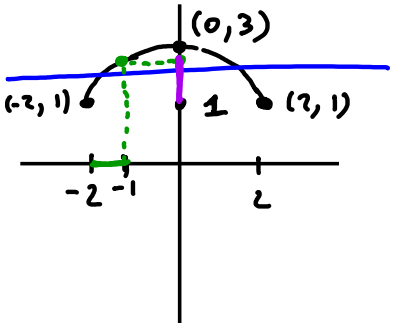
$k = 40 \cdot \frac{15}{2} - \frac{8}{3} \cdot \frac{15^2}{4} = 300 - \frac{450}{3} = 300 - 150 = \boxed{150\ ft^2}$

NAME (First Last):

2. Consider the function $f(x) = 1 + \sqrt{4 - x^2}$ defined on its natural domain $-2 \leq x \leq 2$

(a) Draw the graph of $y = f(x)$.

C (0, 1)



f(-1) = 1+√3

(b) Is f invertible? Justify your answer.

No it does not satisfy the horizontal line test.

(c) Let $g(x) = 1 + \sqrt{4 - x^2}$ on the domain $-2 \leq x \leq -1$ and compute $g^{-1}(x)$, the inverse of g. Show clearly all the step of the calculations you are doing to compute g^{-1}.

$y = 1 + \sqrt{4-x^2}$ $\quad y - 1 = \sqrt{4-x^2}$ $\quad (y-1)^2 = 4 - x^2$

$x^2 = 4 - (y-1)^2$ $\quad x = \pm\sqrt{4-(y-1)^2}$ choose −

(see above picture) $\quad g^{-1}(x) = -\sqrt{4-(x-1)^2}$

(d) What are the domain and range of $g^{-1}(x)$?

Domain: $1 \leq x \leq 1+\sqrt{3}$ (range of g)

Range: $-2 \leq y \leq -1$ (domain of g)

Extra credit : Find all values of d such that the parabola $y = x^2 + dx + d^2$ has vertex on the line $y = 1$

$h = -\frac{d}{2}$ $\quad k = \left(-\frac{d}{2}\right)^2 + d\left(-\frac{d}{2}\right) + d^2 = \frac{d^2}{4} - \frac{d^2}{2} + d^2 = \frac{3}{4}d^2$

We want $k = 1$ so $\frac{3}{4}d^2 = 1$ $\quad d^2 = \frac{4}{3}$ $\quad d = \pm\frac{2}{\sqrt{3}}$