Math 120 - Fall 2022
Final Exam

NAME (First,Last) (same as in Canvas): ....

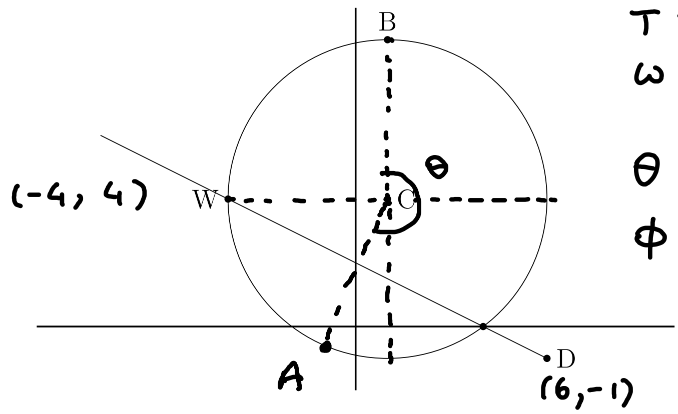
UW email: ....

Student ID ....

Section ....

- You have 2 hrs and 30 min to complete this exam.
- Make sure your writing is clear and dark enough.
- Unless stated otherwise, you **MUST** show work for credit.
- Your work needs to be neat and legible.
- Unless the problem gives you different instructions, you can give exact answers or round off your answers to 2 decimal places.
- The only calculator allowed is the TI 30X IIS. You are allowed two 8x11 sheets of notes, written both sides.
- Box your final answer, when appropriate.
- Raise your hand if you have a question.
- IMPORTANT: when you are done, scan your exam into Gradescope. Please make sure your scan is readable and complete.

1. Bob runs around the circle pictured below. The center of the circle is at C (1,4). It takes Bob 1 hour and 10 minutes to run around the circle once. At time $t = 0$, Bob starts running from some point A on the circle; after 40 minutes he is at B (1, 9); B is the Northernmost point on the circle.



$T = 70$ min

$\omega = \frac{2\pi}{70}$ rad/min

$\theta = \frac{2\pi}{70} \cdot 40 = \frac{8}{7}\pi$

$\phi = \theta - \frac{\pi}{2} = \frac{16}{14}\pi - \frac{7}{14}\pi = \frac{9}{14}\pi$

$r = 9 - 4 = 5$

(a) Find the parametric equation of motion for Bob.

$$\boxed{\begin{aligned} x &= 1 + 5\cos\left(\frac{2\pi}{70}t - \frac{9}{14}\pi\right) \\ y &= 4 + 5\sin\left(\frac{2\pi}{70}t - \frac{9}{14}\pi\right) \end{aligned}}$$

(b) Ann starts running at the same time as Bob.She starts at D and runs along the line DW. W is the Westernmost point on the circle; D has coordinates (6,-1). Ann reaches W at the same time as Bob. Find the parametric equations of motion for Ann.

Bob reaches W at $t = 40 + \frac{70}{4} = \frac{230}{4} = 57.5$ min

$v_x = \frac{-4-6}{57.5}$ $\qquad$ $v_y = \frac{4-(-1)}{57.5}$

$$\boxed{\begin{aligned} x &= 6 - \frac{100}{575}t \\ y &= -1 + \frac{50}{575}t \end{aligned}}$$

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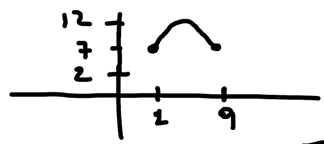
2. Quantity A oscillates sinusoidally between a minimum of 2 and a maximum of 12. Tomorrow (t=1) the value of the quantity will be 7 and raising and it is expected to be down at 7 again 9 days from today (t=9). Find a formula that gives the value of quantity A t days from today.

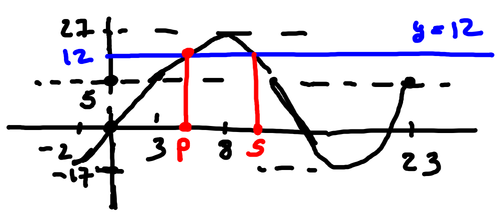


$A = \frac{12-2}{2} = 5 \quad D = \frac{12+2}{2} = 7$

$B = 2(9-1) = 16 \quad C = 1$

$$q(t) = \boxed{5 \sin\left(\frac{2\pi}{16}(t-1)\right) + 7}$$

The value of quantity B t days from today is given by the function $f(t) = 22\sin(\frac{\pi}{10}(t-3)) + 5$. Sam needs quantity B to have a value above 12 for (non consecutive) intervals of time that add up to 30 days. How long does Sam have to wait? (Give your answer in days from today: for example after 42.35 days from today quantity B will have been above 12 for 30 days).



$22\sin\left(\frac{\pi}{10}(t-3)\right) + 5 = 12$

$\sin\left(\frac{\pi}{10}(t-3)\right) = \frac{7}{22}$

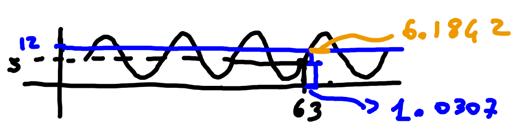
principal $t_1 = 3 + \frac{10}{\pi}\arcsin\left(\frac{7}{22}\right) \approx 4.0307$

symmetry $t_2 = 8 + (8 - 4.0307) \approx 11.9693$

$t_2 - t_1 = 7.9386$

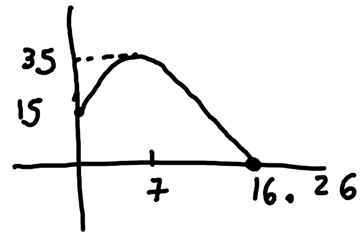
$7.9386 \times 3 = 23.8158$: at day $3 + 20 \cdot 3 = 63$ value above 12 for 23.8158 days. We need an extra 6.1842 days

so $63 + \underbrace{1.0307}_{P=3} + 6.1842 \approx 70.21$ days from today



3. Jack is standing on a cliff 15 feet above the sea. He kicks a ball into the sea. The height of the ball above the sea t seconds after being kicked, is described by a quadratic function h(t). You know that h(0) = 15 and that the ball reaches its maximum height of 35 feet above the sea 7 seconds after being kicked.

(a) Find a formula for h(t).



$h(t) = a(t-7)^2 + 35$

$15 = a \cdot 49 + 35$, $\quad a = -\frac{20}{49}$

$h(t) = -\frac{20}{49}(t-7)^2 + 35$

(b) Find the time t_1 when the ball hits the water.

$-\frac{20}{49}(t-7)^2 + 35 = 0$

$(t-7)^2 = \frac{49}{20} \cdot 35$

$t_1 = 7 \pm \frac{7}{2}\sqrt{7}$

$t_1 = 7 + \frac{7}{2}\sqrt{3} \approx 16.26$ sec

(c) At what time, when the ball is going up, is it 30 feet above the sea?

$-\frac{20}{49}(t-7)^2 + 35 = 30$

$(t-7)^2 = \frac{5 \cdot 49}{20}$

$t = 7 \pm \frac{7}{2}$ $\quad$ going up is $7 - 3.5 = 3.5$ sec

(going down is $7 + 3.5$)

(d) Find a formula for the function $t = g(y)$, that gives you the time when the ball is going up and it is at an height y above the sea level. Give the domain and range for g.

$g(y)$ is the inverse of $h(t)$ on $[0, 7]$

range of $h(t)$ is $[15, 35]$

Domain of g $[15, 35]$

Range of g $[0, 7]$

$y = -\frac{20}{49}(t-7)^2 + 35$

$\frac{20}{49}(t-7)^2 = 35 - y$

$(t-7)^2 = \frac{49}{20}(35-y)$

$t = 7 \pm \sqrt{\frac{49}{20}(35-y)}$ $\quad$ choose $-$ $\quad$ $g(y) = 7 - \sqrt{\frac{49}{20}(35-y)}$

4. At 9 am today the value of a certain stock is \$1 per share. During the morning, that is until 12 pm, the value of the stock increases exponentially, doubling every 1.5 hours, but in the afternoon the value decreases linearly, reaching a value of \$ 2 per share at 5 pm.

(a) Write a multipart formula for the function f(t), that gives you the value of the stock t hours after 9 am.

$$f(t) = \begin{cases} 2^{t/1.5} & 0 \le t \le 3 \\ 4 - \frac{2}{5}(t-3) & 3 \le t \le 8 \end{cases}$$

$(3 \quad 4) \qquad (8 \quad 2)$

$4 + \frac{2-4}{8-3}(t-3)$

(b) Draw a graph of $f(t)$. Mark all relevant points on the graph.

(c) Jack tells you he bought the stock today at \$ 3 per share. Find all times when Jack could have bought the stock.

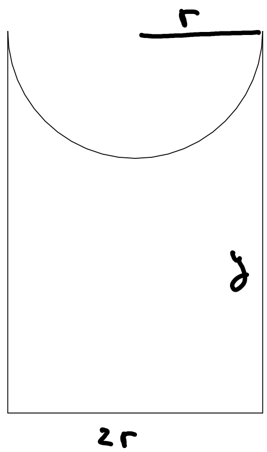
$2^{\frac{t}{1.5}} = 3 \qquad \frac{t}{1.5}\ln 2 = \ln 3 \qquad t = \frac{\ln 3}{\ln 2} \cdot 1.5 \approx 2.3774$

$0.3774 \times 60 = 22.64$ at $\boxed{11:23 \text{ am}}$

OR $\quad 4 - \frac{2}{5}(t-3) = 3 \qquad 1 = \frac{2}{5}(t-3)$

$t = 2.5 + 3 = 5.5$ at $\boxed{2.30 \text{ pm}}$

5. You have 10 feet of wire. First you get z feet of wire and you bend it into a lower semicircle, then you use the rest of the wire to form three sides of a rectangle and you connect to the semicircle as shown in the figure below. What should z be, if you want the region enclosed by the wire to have the maximum possible area ? (Remember that the area of a circle of radius r is πr^2, and the circumference of a circle of radius r is $2\pi r$).



$$A = 2r \cdot y - \frac{1}{2}\pi r^2$$

$$\pi r + 2r + 2y = 10$$

$$y = 5 - \frac{\pi}{2} r - r$$

$$A = -\frac{1}{2}\pi r^2 + 2r\left(5 - \frac{\pi}{2} r - r\right)$$

$$A = \left(-\frac{1}{2}\pi - \pi - 2\right) r^2 + 10r = \left(-\frac{3}{2}\pi - 2\right) r^2 + 10r$$

A is max when $r = \dfrac{-10}{2\left(-\frac{3}{2}\pi - 2\right)} = \dfrac{5}{\frac{3}{2}\pi + 2} \approx 0.7449$

$\therefore z = \pi r \approx 2.34$ ft.