

Math 120 (Pezzoli)
Fall 2019
Midterm #1

Name _____

TA: _____

Section: _____

Instructions:

- Your exam contains 3 problems.
- Your exam should contain 4 pages; please make sure you have a complete exam.
- Box in your final answer when appropriate.
- Unless stated otherwise, you **MUST** show work for credit. No credit for answers only. If in doubt, ask for clarification.
- Your work needs to be neat and legible.
- You are allowed one 8.5×11 sheet of notes (both sides).
- The only calculator allowed is the Ti-30x IIS.
- When you are not giving exact answers, round off your final answers to 2 decimal places.

Problem #1 (15 pts) _____

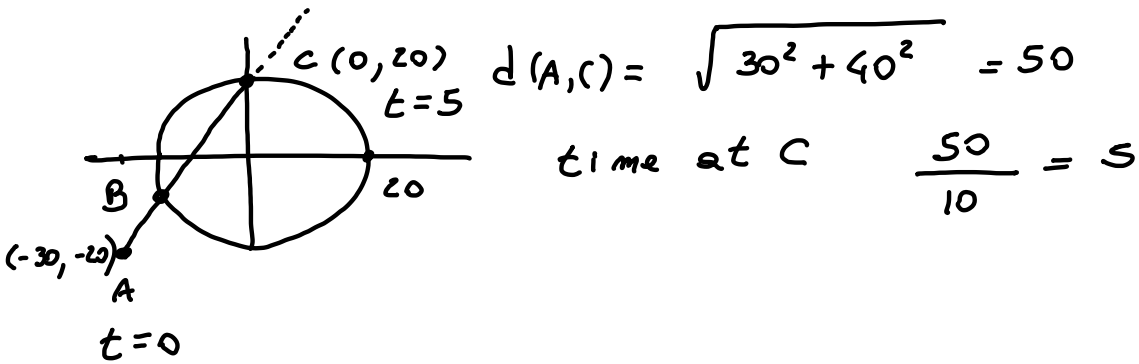
Problem #2 (15 pts) _____

Problem #3 (10 pts) _____

TOTAL (40 pts) _____

1. A rabbit's burrow is located 30 feet to the west and 20 feet to the south of the center of a grass field having the shape of a circle with radius 20 feet. One day the rabbit leaves his burrow at time $t = 0$ and runs in a straight line towards the grass field, crossing it and exiting at its Northernmost edge. The rabbit runs at a constant speed of 10 ft/sec.

(a) Introduce a coordinate system with the origin at the center of the grass field and draw a picture of the grass field and of the rabbit's path. Show the coordinates of the rabbit's burrow and of the Northernmost edge of the grass field.



(b) Find the parametric equations of motion for the rabbit.

$$x = -30 + 6t$$

$$v_x = \frac{30}{5} = 6$$

$$y = -20 + 8t$$

$$v_y = \frac{40}{5} = 8$$

(c) When does the rabbit enter the grass field?

$$(-30 + 6t)^2 + (-20 + 8t)^2 = 20^2$$

$$900 - 360t + 36t^2 + 400 - 320t + 64t^2 = 400$$

$$100t^2 - 680t + 900 = 0$$

$$5t^2 - 34t + 45 = 0$$

$$t = \frac{34 \pm \sqrt{34^2 - 4 \cdot 5 \cdot 45}}{10} = \frac{34 \pm 16}{10} = \frac{50}{10} = 5 \text{ sec}$$

2. Smoke from a fire is spreading into Western Washington. A group of scientists is monitoring air quality and measuring the air quality index. An air quality index between 0 and 50 is considered good. An air quality index above 300 is considered hazardous for the general public, and everybody is advised to remain inside.

(a) Suppose three days ago the air quality index was 180, and today it is 204. Assuming the air quality index follows a linear model, write a function $f(t)$ that gives the air quality index t days from today. What will be the air quality index tomorrow?

$$(-3, 180) \quad (0, 204)$$

$$f(t) = 204 + \frac{204-180}{3} \cdot t$$

$$f(t) = 204 + 8t$$

$$f(1) = 212$$

(b) Suppose instead yesterday air quality index was 180 and the quality index is going to reach its worse (highest) value of 252 in two days. Assuming a quadratic model, in how many days (from today) will the air quality index reach 52?

$$(-1, 180) \quad (2, 252) \text{ vertex}$$

$h \quad k$

$$f(t) = a(t-2)^2 + 252$$

$$180 = a \cdot 9 + 252 \quad a = -8$$

$$f(t) = -8(t-2)^2 + 252$$

$$52 = -8(t-2)^2 + 252$$

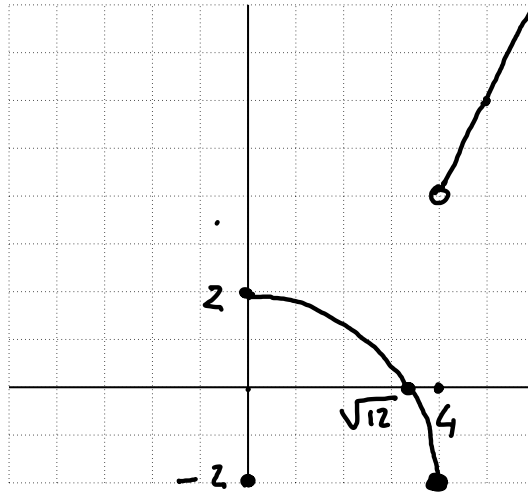
$$8(t-2)^2 = 200 \quad ; \quad (t-2)^2 = \frac{200}{8} = 25 \quad t = 2 \pm 5$$

$$t = 7$$

3. In this problem give exact answers. The function f is defined as follows :

$$f(x) = \begin{cases} \sqrt{16-x^2} - 2, & \text{if } 0 \leq x \leq 4 \\ 2x - 4, & \text{if } x > 4 \end{cases}$$

(a) Draw the graph of f .



(b) Find the y intercept, and mark it on the graph you drew in part (a).

$$f(0) = \sqrt{16} - 2 = \boxed{2}$$

(c) Find the range of f .

$$[-2, 2] \text{ and } (4, \infty)$$

(d) Find the x intercepts and mark them on the graph you drew in part (a).

$$\sqrt{16-x^2} - 2 = 0$$

$$\sqrt{16-x^2} = 2$$

$$16-x^2 = 4$$

$$12 = x^2$$

$$\pm \sqrt{12} = x$$

$$\boxed{x = \sqrt{12}}$$

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