Math 120 - Fall 2019
Final Exam

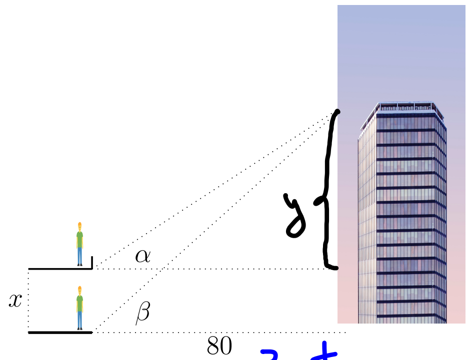
Name: ____________________ Student ID no. : ____________

Signature: ________________________ Section: ________

1	10	
2	17	
3	10	
4	10	
5	13	
6	15	
Total	75	

- Your exam contains 6 problems.
- Your exam should contain 7 pages; please make sure you have a complete exam.
- Box in your final answer when appropriate.
- Unless stated otherwise, you **MUST** show work for credit. No credit for answers only. If in doubt, ask for clarification.
- Your work needs to be neat and legible.
- You are allowed one 8.5×11 sheet of notes (both sides). The only calculator allowed is Texas Instruments TI 30x iis.
- Round off your final answers to 2 decimal places, unless the problem gives you different instructions..

1. Tom is standing on his deck, at a height x from the ground and 80 feet away from a building, directly in front of him. From his position on the deck he measures that the top of the building forms an angle $\alpha = 52°$ with the horizontal. He then goes down to the garden and from a position on the ground directly below his previous position on the deck he measures that the top of the building forms an angle $\beta = 55°$ with the horizontal. What is the height x of Tom 's deck?



3 pt   3 pt

$$\tan(\alpha) = \frac{y}{80}, \quad \tan(\beta) = \frac{y+x}{80}$$

$$\tan\beta = \frac{y}{80} + \frac{x}{80} \qquad \tan\beta = \tan\alpha + \frac{x}{80}$$

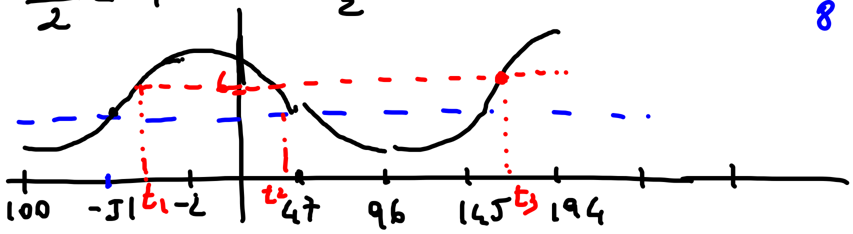
$$x = 80\left(\tan\left(\frac{55\cdot 2\pi}{360}\right) - \tan\left(\frac{52\cdot 2\pi}{360}\right)\right) \approx 11.86 \text{ feet}$$

4 pt for solution

2. The value of stock XX follows a sinusoidal pattern. Two days ago (t=-2) stock XX reached its maximum value of \$ 9. The previous minimum of \$1 was 100 days ago (t=-100). Approximately for how many days in the next 6 months (that is for $0 \leq t \leq 180$) will the stock be worth \$6 or more ? (Round to the nearest integer)

$A = \frac{9-1}{2} = 4 \quad D = \frac{9+1}{2} = 5 \quad B = 2\cdot(98) = 196 \,, \quad C = -2-49 = -51$

8 pt



$$4 \sin\left(\frac{2\pi}{196}(t+51)\right) + 5 = 6$$ 1 pt

$$\sin\left(\frac{\pi}{98}(t+51)\right) = \frac{1}{4}$$

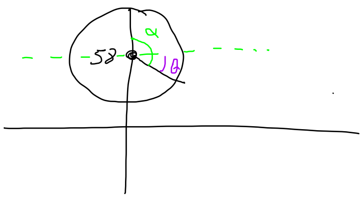
$$t_1 = \frac{98}{\pi}\sin^{-1}\left(\frac{1}{4}\right) - 51 \approx -43.1178$$ 2 pt

$$t_2 = -2 + (-2 - t_1) = 39.1178$$ 2 pt

$$t_3 = t_1 + 196 = 152.8822$$ 2 pt

Answer is $t_2 + (180 - t_3) = 66$ 2 pt

3. Tom is riding a ferris wheel. The center of the wheel is 58 feet above the ground. One complete revolution of the wheel takes 4 min. The wheel rotates counterclockwise. It takes Tom 1.6 min. to reach the top of the wheel from the position where he boards the wheel and from the position where he boards the wheel to the top of the wheel Tom travels a distance of 44 π feet. Find Tom's height above the ground 2.2 min after he boards the wheel.



$\omega = \frac{2\pi}{4} = \frac{\pi}{2}$ red/min

$\alpha = \frac{\pi}{2} \cdot 1.6 = 0.8\pi$

$\theta = \alpha - \frac{\pi}{2} = 0.3\pi$

$44\pi = 0.8\pi \cdot r \qquad r = \frac{44}{0.8} = 55$

$y = 58 + 55 \sin\left(\frac{\pi}{2} t - 0.3\pi\right)$ **8 pt**

$y(2.2) = 58 + 55 \sin(1.1\pi - 0.3\pi) = 90.33$ feet **2pt**

4. The price of a single bottle of wine is a linear function of the total number x of bottles produced. If $x = 100$ the price of a bottle is \$ 90. If $x = 300$ the price of a bottle is \$70. How many bottles should a winery produce in order to maximize gross revenue (that is the amount of money the winery gets from selling the bottles it produces) ? You can assume the winery can sell all the bottles it produces.

$(100, 90) \quad (300, 70)$

$p(x) = 90 - \frac{20}{200}(x-100) = 90 - \frac{1}{10}(x-100)$ 3 pt

3 pt

$r(x) = x \cdot p(x) = x \cdot \left(90 - \frac{1}{10}(x-100)\right) = 90x - \frac{1}{10}x^2 + 10x = -\frac{1}{10}x^2 + 100x$

$\cap$ max at vertex $\quad h = \frac{-100}{-\frac{2}{10}} = 500$

1 pt

3 pt

5. The value of stock AA doubles every 10 years. Stock BB increases 15% every 3 years. This year (2019) the stocks have the same value. When will the value of stock AA be twice the value of stock BB ? (Give the answer in years, i.e in 2040)

4pt $f(t) = A_0 \sqrt[10]{2}^{\,t}$ value of stock XX t years after 2019

4pt $g(t) = A_0 \sqrt[3]{1.15}^{\,t}$ value of stock XY " "

want $f(t) = 2\, g(t)$ 2pt

$$A_0 \sqrt[10]{2}^{\,t} = 2 A_0 \sqrt[3]{1.15}^{\,t}$$

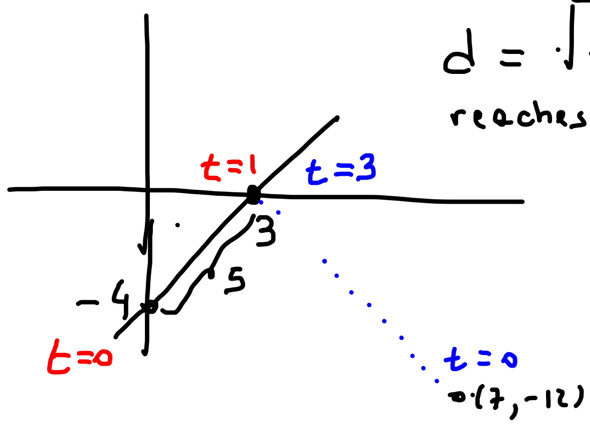
$$\left(\frac{\sqrt[10]{2}}{\sqrt[3]{1.15}}\right)^t = 2$$

$$t = \frac{\ln 2}{\ln\left(\frac{\sqrt[10]{2}}{\sqrt[3]{1.15}}\right)} \approx 30$$

in $\boxed{2049}$ 3pt

6. Bob and Ann are moving at constant speeds in the xy plane.1 unit on the x and y axes corresponds to 1 foot. They start moving at the same time $t=0$. Ann starts at the point $(0,-4)$ and moves along the line $y=-4+\frac{4}{3}x$ towards the x axis at a speed of 5 feet/sec. Bob starts at (7,-12) and passes through the same point on the x axis as Ann, but 2 sec after she does. Find the distance between Bob and Ann 9 sec after they start moving.

$y=-4+\frac{4}{3}x$ crosses the x axis at $(3,0)$; to reach $(3,0)$ Ann has to travel a distance



$d=\sqrt{3^2+4^2}=5$, so she reaches $(3,0)$ at time $t=\frac{5}{5}=1$. Bob reaches $(3,0)$ at $t=3$

Ann 5pt $\quad x = 0 + 3t$
$\quad y = -4 + 4t$

Bob $\quad x = 7 - \frac{4}{3}t$
$\quad y = -12 + 4t$ 5pt

at $t=9$ Ann $(27, 32)$
Bob $(-5, 24)$

$$d(9) = \sqrt{32^2 + 8^2} = 32.98 \text{ feet}$$