Math 120 C - Autumn 2017
Midterm Exam Number Two
November 16th, 2017

Name: Saul Uxin Student ID no. : 8675309

Signature: [signature] Section: CE

1	15	15
2	15	15
3	15	15
4	15	15
Total	60	60

Wow, great work!

- This exam consists of FOUR problems on FIVE pages, including this cover sheet.
- Show all work for full credit.
- You may use a TI-30X IIS calculator during this exam. Other calculators and electronic device are not permitted.
- You do not need to simplify your answers.
- If you use a trial-and-error or guess-and-check method when a more rigorous method is available, you will not receive full credit.
- If you write on the back of the page, please indicate that you have done so!
- Draw a box around your final answer to each problem.
- You may use one hand-written double-sided 8.5" by 11" page of notes.
- You have 50 minutes to complete the exam.

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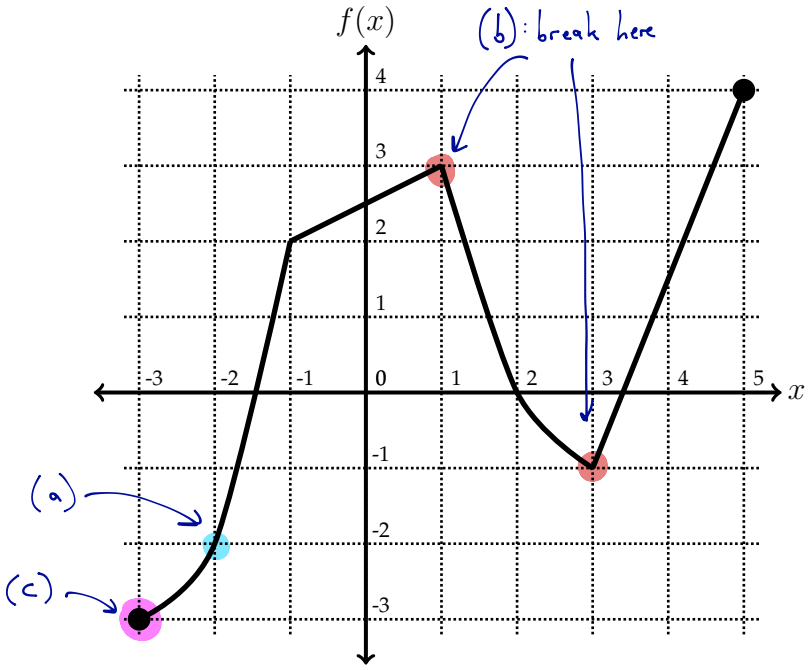
1. **[5 points per part]** Hey, check out this awesome graph!



Use the graph to answer the following questions.

(a) Find $f(f(f(f(f(f(f(f(f(f(-2))))))))))$. Explain, briefly.

-2, -2, -2, ⋮

-2, because -2 is a fixed point.

(b) Oh no, f isn't one-to-one!
Please break its domain into three intervals so that f is one-to-one on each piece.

$[-3, 1], (1, 3], (3, 5]$

(, [, whatever...

(c) Take the graph of $y = f(x)$. Shift it 3 units to the right, then scale it vertically by a factor of 2, then shift it 5 units down. Let the result be $y = g(x)$.
Find $g(0)$.

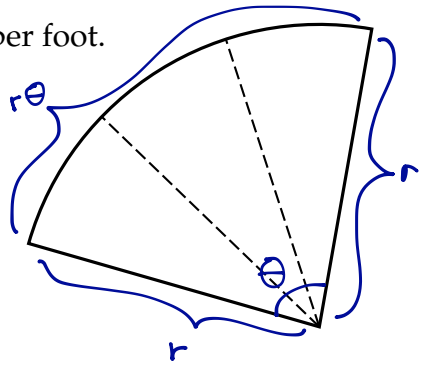
$y = f(x)$
↓ right 3
$y = f(x-3)$
↓ scale vert. by 2
$\frac{y}{2} = f(x-3)$
↓ down 5
$\frac{y+5}{2} = f(x-3)$
↓
$\frac{g(x)+5}{2} = f(x-3)$
↓
$\frac{g(0)+5}{2} = f(0-3)$

$g(0) = 2f(-3) - 5$
$= 2(-3) - 5$
$g(0) = \boxed{-11}$

2. **[15 points]** I have $4000 with which I'd like to build a fence in the shape of a sector, with two internal partitions each running from the arc to the opposite corner, as shown below.

   The outside fence costs $5 per foot, and the partitions cost $3 per foot.

   What is the **maximum possible total area** inside the fence?



$\text{Area} = \frac{1}{2} r^2 \theta$

$\text{Cost} = 5(2r + \theta r) + 3(2r) = 4000$

outside inside

$16r + 5\theta r = 4000$

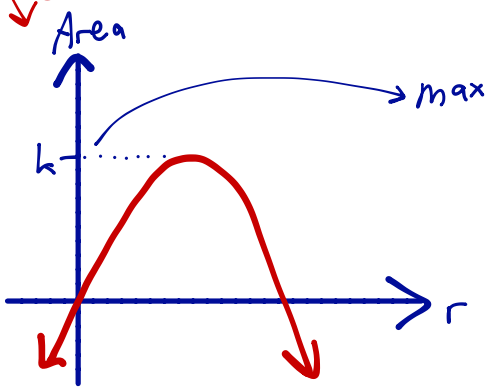
$16 + 5\theta = \frac{4000}{r}$

$\theta = \frac{800}{r} - \frac{16}{5}$

$\text{Area} = \frac{1}{2} r^2 \left(\frac{800}{r} - \frac{16}{5} \right)$

$= 400r - \frac{8}{5} r^2$

graph!



max area $= 0 - \frac{(400)^2}{4\left(\frac{-8}{5}\right)}$

$= 25000$ sq. ft.

3. Here are some facts about $f(x)$, a linear-to-linear rational function:

- The domain of f is $(-\infty, 2) \cup (2, \infty)$.
- The graph $y = f(x)$ has a y-intercept of 5.
- $f(10) = 3$.

(a) **[8 points]** Write a formula for $f(x)$.

$f(x) = \frac{ax+b}{x+d}$

vert. asymptote $x=2$ → $d=-2$

$f(0)=5$ → $\frac{b}{d}=5$ → $b=5d$ → $b=-10$

$3=\frac{10a+b}{10+d}$ → $30+3d=10a+b$ → $30-6=10a-10$ → $a=3.4$

$$f(x)=\frac{3.4x-10}{x-2}$$

(b) **[7 points]** Write a formula for $f^{-1}(x)$.

$y=\frac{3.4x-10}{x-2}$

$y(x-2)=3.4x-10$

$xy-2y=3.4x-10$

$xy-3.4x=2y-10$

$x(y-3.4)=2y-10$

$x=\frac{2y-10}{y-3.4}$

$$f^{-1}(x)=\frac{2x-10}{x-3.4}$$

4. **[5 points per part]**

Some bad things happened to the moon, and now it's in several pieces. The number of pieces is an exponential function of time.

Right now, there are 7 pieces.

In five months, there will be 130 pieces.

(a) Write a function $f(t)$ for the number of pieces t months from now.

$f(t) = A_0 b^t$

$A_0 = 7$

$f(5) = 7b^5 = 130$

$b^5 = \frac{130}{7}$

$b = \left(\frac{130}{7}\right)^{1/5}$

$f(t) = 7\left(\left(\frac{130}{7}\right)^{1/5}\right)^t$

$f(t) = 7\left(\frac{130}{7}\right)^{\frac{t}{5}}$

(b) When will there be 1 million pieces?

$1000000 = 7\left(\frac{130}{7}\right)^{\frac{t}{5}}$

$\ln(1000000) = \ln(7) + \frac{t}{5}\ln\left(\frac{130}{7}\right)$

$\frac{t}{5}\ln\left(\frac{130}{7}\right) = \ln(1000000) - \ln(7)$

$t = \frac{5(\ln(1000000) - \ln(7))}{\ln\left(\frac{130}{7}\right)} \approx 20.313$ months from now.

(c) The mass of the whole moon is 73 yottagrams.

Write a function $g(t)$ for the average mass of each piece, t years from now.

Write your answer in standard exponential form.

(Leave your answer in yottagrams. Don't worry about what a yottagram is.)

$g(t) = \frac{\text{mass of moon}}{\#\text{ of pieces}} = \frac{73}{7\left(\frac{130}{7}\right)^{\frac{12t}{5}}}$ because t years $= 12t$ months

$= \frac{73}{7}\left(\frac{1}{\left(\frac{130}{7}\right)^{\frac{12t}{5}}}\right) = \frac{73}{7}\left(\left(\frac{7}{130}\right)^{\frac{12}{5}}\right)^t$