Math 120 C - Autumn 2017
Midterm Exam Number One
October 19th, 2017

Name: _____ Student ID no. : _____

Signature: _____ Section: _____

1	15	
2	15	
3	15	
4	15	
Total	60	

- This exam consists of FOUR problems on FIVE pages, including this cover sheet.
- Show all work for full credit.
- You may use a TI-30X IIS calculator during this exam. Other calculators and electronic device are not permitted.
- You do not need to simplify your answers.
- If you use a trial-and-error or guess-and-check method when a more rigorous method is available, you will not receive full credit.
- If you write on the back of the page, please indicate that you have done so!
- You may use one hand-written double-sided 8.5" by 11" page of notes.
- You have 50 minutes to complete the exam.

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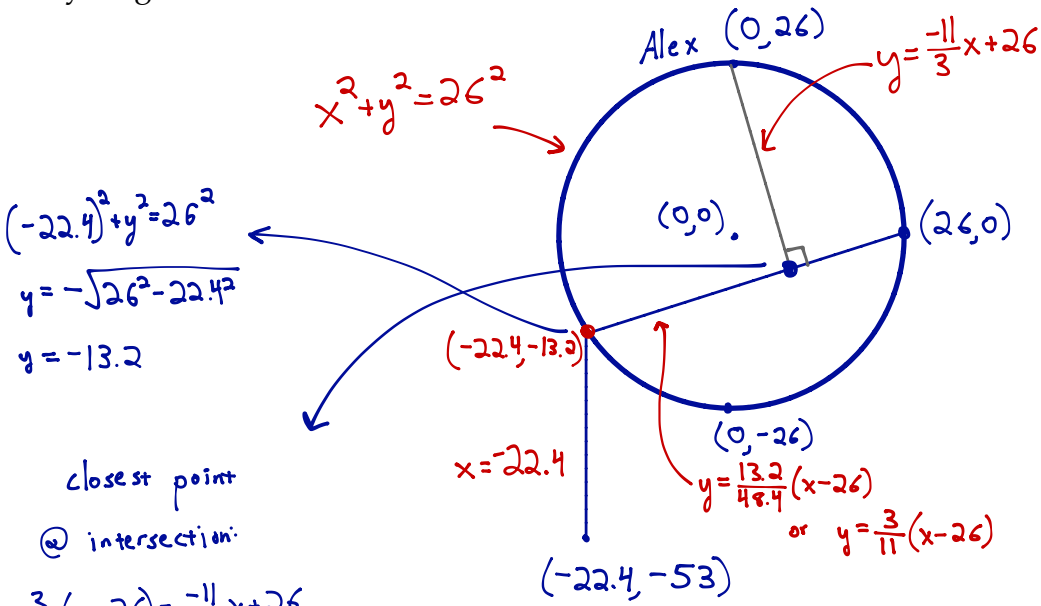
1. **[15 points]** Alex is at the *northernmost* point of a circular parking lot with radius 26 feet.

   Dylan stands 22.4 feet west and 27 feet south of the *southernmost* point of the parking lot.

   Dylan walks due north until he hits the edge of the parking lot. Then, he turns and walks in a straight line towards the *easternmost* point of the parking lot.

   How close does Dylan get to Alex?



$(-22.4)^2+y^2=26^2$

$y=-\sqrt{26^2-22.4^2}$

$y=-13.2$

closest point @ intersection:

$\frac{3}{11}(x-26)=\frac{-11}{3}x+26$

$\frac{3}{11}x-\frac{78}{11}=\frac{-11}{3}x+26$

$x=8.4$

$y=-4.8$

Dist. from $(8.4,-4.8)$ to $(0,26)$:

$\sqrt{(8.4-0)^2+(-4.8-26)^2}\approx 31.925$ ft

2. **[5 points per part]** Chidi and Tahani are walking around the coordinate plane.

   Chidi begins at the point $(6, -4)$ and walks towards $(-2, 2)$ in a straight line at constant speed, reaching it in 10 seconds.

   Tahani begins at the point $(5, 8)$ and also walks in as straight line at constant speed.

   One second after Chidi crosses the y-axis, Tahani also crosses the y-axis at the same place.

   (a) Write parametric equations for Chidi's coordinates after t seconds.

$x_0 = 6 \quad y_0 = -4$

$x_1 = -2 \quad y_1 = 2$

$\Delta x = -8 \quad \Delta y = 6$

$\Delta t = 10$

$x = 6 - \frac{8}{10}t$

$y = -4 + \frac{6}{10}t$

or

$x = 6 - \frac{4}{5}t$

$y = -4 + \frac{3}{5}t$

   (b) *Where* and *when* does Chidi cross the y-axis?

$x = 6 - \frac{4}{5}t = 0$

$t = 7.5$

$y = -4 + \frac{3}{5}(7.5) = 0.5$

$(0, 0.5)$

   (c) What is Tahani's speed?

dist. from $(5, 8)$ to $(0, 0.5)$

$$\text{Speed} = \frac{\text{dist}}{\text{time}} = \frac{\sqrt{5^2 + 7.5^2}}{8.5} \approx 1.06 \text{ units/sec}$$

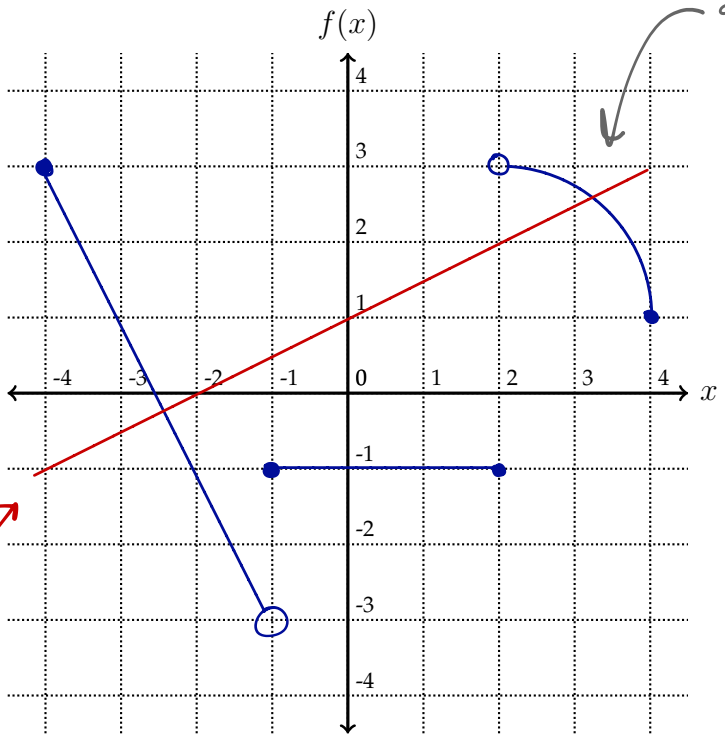
1 second longer than Chidi takes to reach y-axis.

3. Consider the following multipart function f:

$$f(x) = \begin{cases} -2x - 5 & \text{if } -4 \le x < -1 \\ -1 & \text{if } -1 \le x \le 2 \\ 1 + \sqrt{4 - (x-2)^2} & \text{if } 2 < x \le 4 \end{cases}$$

(a) **[7 points]** Sketch a graph of $f(x)$ below.

right-half of an upper semicircle



(b) **[8 points]** Find all values of x such that $f(x) = \frac{1}{2}x + 1$.

from this red line we expect two answers.

if $-4 \le x < -1$

$-2x - 5 = \frac{1}{2}x + 1$

$-2.5x = 6$

$x = -2.4$

if $-1 \le x \le 2$

$-1 = \frac{1}{2}x + 1$

$x = -4$

Nope

if $2 < x \le 4$

$1 + \sqrt{4 - (x-2)^2} = \frac{1}{2}x + 1$

$4 - (x-2)^2 = \frac{1}{4}x^2$

$0 = \frac{5}{4}x^2 - 4x$

$0 = x\left(\frac{5}{4}x - 4\right)$

$x = 0$ Nope or $x = 3.2$

So: $x = -2.4$ or 3.2

4. The temperature in Paraboland is modeled by a quadratic function of time.

   Right now, the temperature is $20°$.

   In 2 days, the temperature will be $27°$.

   In 10 days, the temperature will be $51°$.

   (a) **[12 points]** Write a function $f(x)$ for the temperature in Paraboland x days from now.

$f(x) = ax^2 + bx + c$

$f(0) = 20 \rightarrow 20 = c$

$f(2) = 27 \rightarrow 27 = 4a + 2b + c \rightarrow -5(7 = 4a + 2b)$

$f(10) = 51 \rightarrow 51 = 100a + 10b + c \rightarrow +31 = 100a + 10b$

$-4 = 80a$

$a = \frac{-4}{80}$

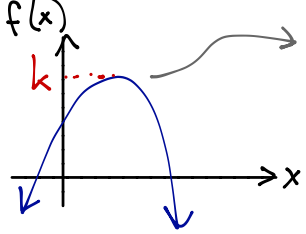
$a = -0.05$

$7 = 4(-0.05) + 2b$

$b = 3.6$

$f(x) = -0.05x^2 + 3.6x + 20$

   (b) **[3 points]** What will the maximum temperature be?



$k = c - \frac{b^2}{4a} = 20 - \frac{(3.6)^2}{4(-0.05)} =$ **84.8°**