## Math 120

## **Final Examination**

Your Name	Your Signature	
Student ID #	Quiz Section	
Professor's Name	TA's Name	

- Turn off all cell phones, pagers, radios, mp3 players, and other similar devices.
- Round your final answers to two points after the decimal unless otherwise instructed.
- This exam is closed book. You may use one  $8.5'' \times 11''$  sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.
- You can use a Texas Instruments TI-30X IIS calculator. No other calculators are permitted.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Place a box around your answer to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 8 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	

Question	Points	Score
5	10	
6	10	
7	10	
8	10	
Total	80	

1. (10 points) Patrick is using a quadratic model to compute your grade. He wants to have a score of 40 on the exam give a grade of 2.0. A score of 70 on the exam gives a grade of 3.0, and a score of 90 gives a grade of 4.0.

Find the formula of the quadratic function g(x) that computes the grade g corresponding to the exam score x.

What exam score gives a grade of 3.6?

- 2. Let  $g(x) = e^{-3x}$  and h(x) = 1 2x.
  - (a) (5 points) Compute f(x) = h(g(x)) and its inverse  $f^{-1}(x)$ . State the domains and ranges of both f(x) and its inverse  $f^{-1}(x)$ .

(b) (5 points) Sketch the graph of the function y = f(x) by first graphing y = g(x) and y = -2g(x) on the axes provided below. Label the *y*-intercepts and show, if any, asymptotes. You don't have to use the same scaling in the three graphs.



3. Tom and Jack are moving in the *xy*-plane along straight lines at constant speeds. They start moving at the same time.

Tom starts from the point (12,0) and heads directly toward the point (-8,8), reaching it in 12 seconds. Jack starts from the point (6,13) and travels along the line y = x+7. Jack moves toward the *y*-axis. It takes Jack half as long to reach the *y*-axis as it takes Tom to reach the *y*-axis.

(a) (3 points) Find the parametric equations of motion for Tom.

(b) (3 points) Find the parametric equations of motion for Jack.

(c) (4 points) How long have they been moving when the distance between Tom and Jack is as small as it ever gets?

4. (10 points) The population of raccoons in the U District is a sinusoidal function of time. The population oscillates between 20 and 80 raccoons. In 2007, there were 50 raccoons. The population then decreased, reaching the minimum, 20 raccoons, in 2009.

For how long between the years 2006 and 2015 is the population of raccoons more than 65?

5. (10 points) Winfield is measuring trees in a forest. Standing on the flat ground exactly half-way between two trees, he measures the angle of elevation the top of each tree. One angle is 64°, and the other is 78°. If one tree is 50 feet taller than the other, how far apart (horizontal distance along the ground) are the two trees?

Note: The angle of elevation is the angle between the ground and the line joining the point where Winfield is standing with the top of the tree.

- 6. A sheet of metal is in the shape shown below. The lower left angle is a right angle as shown.
  - (a) (5 points) You make a vertical cut x centimeters from the left edge of the sheet. Write down a multi-part function A(x) giving the area to the left of the cut as a function of x.



(b) (5 points) You make a vertical cut so that you end up with two pieces of equal area. Where do you make the cut?

7. (10 points) In the year 395 the Roman Empire split into two parts, with capital cities of Rome in the West and Constantinople in the East. At the time of the split, the population of Rome was 35,000 and the population of Constantinople was 27,000. By the year 476, the population of Rome had reached 41,500. In 1000, the population of Constantinople was twice the population of Rome.

If we assume that the population of both cities satisfied the exponential model during the times in the question, in which year was their populations the same?

Use **four digits** after the decimal for the constants in your equations.

8. Two wheels of radius 12 cm are mounted on a table. Wheel 1 is on the left and Wheel 2 is on the right. Wheel 1 is connected to an inner wheel with radius 3 cm. The inner wheel on Wheel 2 has radius 5 cm. The inner wheels are connected by a belt. The centers of both wheels are 15 cm above the top of the table and the poles holding them are 50 cm apart.

Before the wheels start to turn, a Dot is marked on the lowest point of Wheel 1 and a Star is marked on the lowest part of Wheel 2, as shown below.

Wheel 2 starts turning at a constant rate of 1 rotation every 18 seconds.



Both the Dot and the Star are in lowest position at time t=0

- (a) (3 points) How high is the point Star measured from the top of the table at t = 1.5 seconds?
- (b) (7 points) What is the distance between the Dot and the Star when the Star reaches its highest point for the first time?