Math 120 C - Autumn 2014
Midterm Exam Number Two
November 13, 2014

Name: _____ Student ID no. : _____

Signature: _____ Section: _____

1	15	
2	15	
3	15	
4	15	
Total	60	

- This exam consists of FOUR problems on FIVE pages, including this cover sheet.
- Show all work for full credit.
- You may use a scientific calculator during this exam. Graphing calculators are not permitted. Also, other electronic devices are not allowed, and should be turned off and put away for the duration of the exam.
- You do not need to simplify your answers.
- If you use a trial-and-error or guess-and-check method when a more rigorous method is available, you will not receive full credit.
- If you write on the back of the page, please indicate that you have done so!
- You may use one hand-written double-sided 8.5" by 11" page of notes.
- You have 50 minutes to complete the exam.

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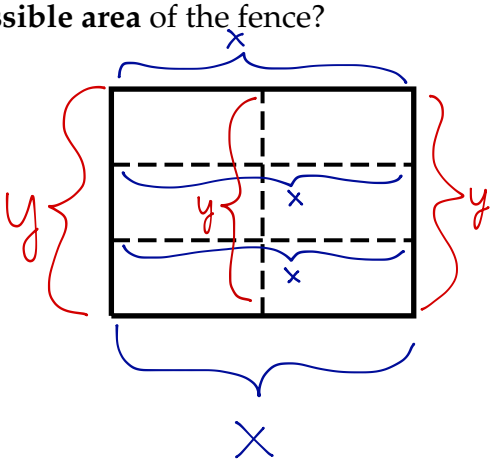
1. **[15 points]** You would like to build a rectangular fence which will be divided into six sections in a 3×2 configuration, as shown below.

   The material for the outside fence costs \$20 per foot, and the material for the inner partitions costs \$5 per foot.

   If you have \$7200 total to spend, what is the **maximum possible area** of the fence?

Let x and y be the lengths of the sides of the whole fence, as shown:



Maximize: $\text{Area} = xy$ ← Two variables! Need some constraint.

Constraint: \$7200 = total cost

$= \$20(\text{perimeter}) + \$5(\text{partition length})$

$= 20(2x+2y) + 5(2x+y)$

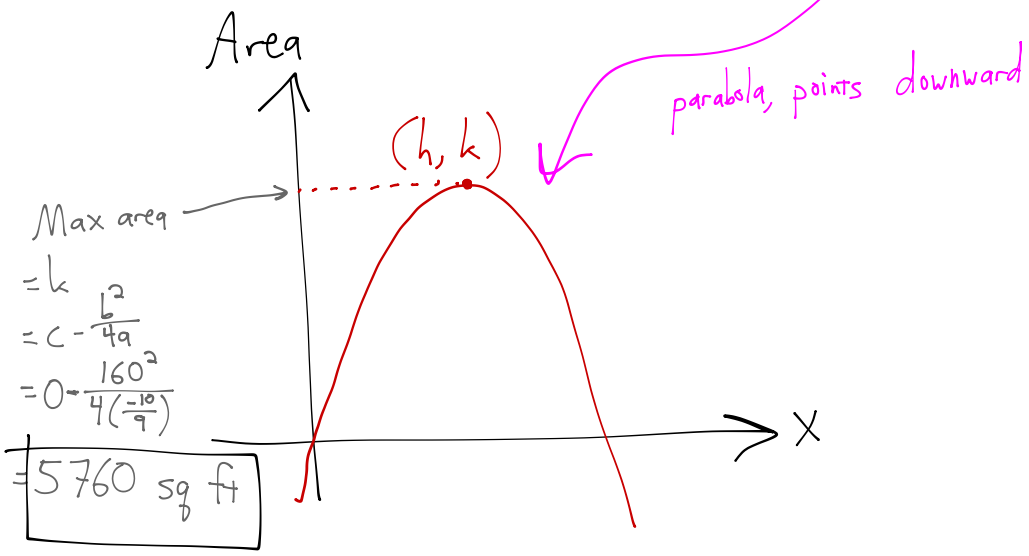
↓ expand, simplify

$7200 = 50x + 45y$ → solve for y

$y = \frac{7200-50x}{45}$

↓ plug into area

$\text{Area} = x\left(\frac{7200-50x}{45}\right) = \frac{-50}{45}x^2 + \frac{7200}{45}x = \frac{-10}{9}x^2 + 160x$



$= k$

$= c - \frac{b^2}{4a}$

$= 0 - \frac{160^2}{4\left(\frac{-10}{9}\right)}$

$= 5760$ sq ft

2. Abbi is downloading her favorite TV show online, but it's taking a while. In fact, the percentage of the show that's been downloaded after t minutes is a linear-to-linear rational function of t.

   At first, of course, the download is 0% complete.

   After 30 seconds, the download is 5% complete.

   After 2 minutes, the download is 16% complete.

   (a) **[13 points]** How much of the show has been downloaded after 30 minutes?

Want a function $f(x) = \frac{ax+b}{x+d}$ so that:

$f(0) = 0 \Rightarrow 0 = \frac{0+b}{0+d} \Rightarrow \boxed{b = 0}$

$f\left(\frac{1}{2}\right) = 5 \Rightarrow 5 = \frac{\frac{1}{2}a + b}{\frac{1}{2} + d} \Rightarrow \frac{5}{2} + 5d = \frac{1}{2}a \Rightarrow a = 5 + 10d$

$f(2) = 16 \Rightarrow 16 = \frac{2a+b}{2+d} \Rightarrow 32 + 16d = 2a \Rightarrow 32 + 16d = 2(5 + 10d)$

$\Downarrow$

$22 = 4d$

$\boxed{d = 5.5}$

$a = 5 + 10d$

$\Downarrow$

$\boxed{a = 60}$

$f(x) = \frac{60x}{x + 5.5}$

In 30 min:

$f(30) = \frac{60(30)}{30 + 5.5} \approx \boxed{50.7\%}$

   (b) **[2 points]** Will the download ever finish? Explain.

Nope! There's a horizontal asymptote at $y = a = 60$, so it won't ever reach 100%.

3. **[5 points per part]**

(a) Let $f(x) = 4^{2x-3}$. Compute $f(f(2))$.

$f(f(2)) = f\left(4^{2(2)-3}\right) = f(4^1) = f(4) = 4^{2(4)-3} = 4^5 = \boxed{1024}$

(b) Let $f(x) = 4^{2x-3}$. Write a formula for the inverse function, $f^{-1}(x)$.

$y = 4^{2x-3}$

$\log_4(y) = 2x-3$

$x = \frac{\log_4(y)+3}{2} \Rightarrow \boxed{f^{-1}(x) = \frac{\log_4(x)+3}{2}}$

(c) Let $f(x) = 4^{2x-3}$. Explain how to turn the graph of $y = 4^x$ into the graph of $f(x)$.
(Do scratch work here, then fill in the blanks below.)

$y = 4^x \xrightarrow{\text{replace } x \text{ w/ } x-3} y = 4^{x-3} \xrightarrow{\text{replace } x \text{ with } 2x, \text{ or } \frac{x}{\frac{1}{2}}} y = 4^{2x-3}$

(shift right 3) (scale horizontally by $\frac{1}{2}$)

Fill in the blanks:

- First, you shift right 3 units.
- Then, you scale horizontally by a factor of $\frac{1}{2}$.

4. **[5 points per part]**

(a) The city of Doublin has a population that doubles every 25 years. In the year 2000, its population was 6000.

Write a function $f(t)$ for the population of Doublin, t years after 2000.

$f(t) = A_0 b^t$

$A_0 = 6000$

$b^{25} = 2$

$b = \sqrt[25]{2}$

$f(t) = 6000\left(\sqrt[25]{2}\right)^t$ or $f(t) = 6000(1.028114)^t$

(b) The city of Tripli has a population that triples every 30 years. In the year 2016, its population was 4000 less than Doublin's.

Write a function $g(t)$ for the population of Tripli, t years after 2000.

$g(t) = A_0 b^t$

$g(16) = f(16) - 4000$

$A_0\left(\sqrt[30]{3}\right)^{16} = 6000\left(\sqrt[25]{2}\right)^{16} - 4000$

$A_0 \approx 2978$

$b^{30} = 3$

$b = \sqrt[30]{3}$

$g(t) = 2978\left(\sqrt[30]{3}\right)^t$ or $g(t) = 2978(1.037299)^t$

(c) When will the population of Tripli be twice the population of Doublin? Round your answer to the nearest year.

$g(t) = 2 \cdot f(t)$

$2978\left(\sqrt[30]{3}\right)^t = 2\left(6000\left(\sqrt[25]{2}\right)^t\right)$

$\left(\sqrt[30]{3}\right)^t = \frac{12000}{2978}\left(\sqrt[25]{2}\right)^t$

$\ln\left(\sqrt[30]{3}\right)^t = \ln\left(\frac{12000}{2978}\left(\sqrt[25]{2}\right)^t\right)$

$\ln\left(\sqrt[30]{3}\right)t = \ln\left(\frac{12000}{2978}\right) + \ln\left(\sqrt[25]{2}\right)t$

$\left(\ln\left(\sqrt[30]{3}\right) - \ln\left(\sqrt[25]{2}\right)\right)t = \ln\left(\frac{12000}{2978}\right)$

$t = \frac{\ln\left(\frac{12000}{2978}\right)}{\ln\left(\sqrt[30]{3}\right) - \ln\left(\sqrt[25]{2}\right)} \approx 156.7$ years after 2000

So, around the year 2157