Math 120 C - Autumn 2014
Midterm Exam Number One
October 16, 2014

Name: _____ Student ID no. : _____

Signature: _____ Section: _____

1	15	
2	15	
3	15	
4	15	
Total	60	

- This exam consists of FOUR problems on FIVE pages, including this cover sheet.
- Show all work for full credit.
- You may use a scientific calculator during this exam. Graphing calculators are not permitted. Also, other electronic devices are not allowed, and should be turned off and put away for the duration of the exam.
- You do not need to simplify your answers.
- If you use a trial-and-error or guess-and-check method when a more rigorous method is available, you will not receive full credit.
- If you write on the back of the page, please indicate that you have done so!
- You may use one hand-written double-sided 8.5" by 11" page of notes.
- You have 50 minutes to complete the exam.

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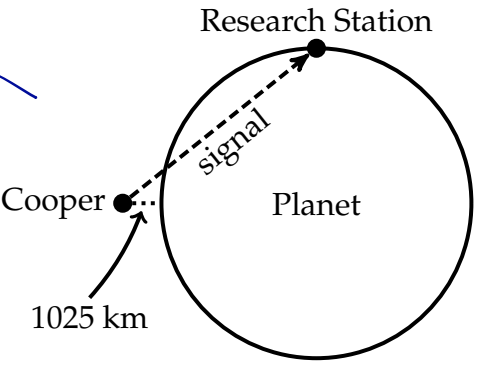
1. **[15 points]** Cooper's spaceship is hovering over the equator of a distant planet with a radius of 4100 kilometers. His ship is 1025 kilometers from the ground. At the north pole of the planet is a research station.

   Cooper sends a radio signal to the research station. It travels in a straight line: first through the atmosphere at a speed of 200 kilometers per millisecond, and then through the planet at a speed of 100 kilometers per millisecond. **How long, in milliseconds**, does the radio signal take to arrive at the station?



New picture

$y = \frac{4100}{5125}(x-0)+4100$

$x^2+y^2=4100^2$

(0, 4100)

(-5125, 0)

(-4100, 0)

(0, 0)

Intersection?

$y = \frac{4100}{5125}x + 4100$

simplifies

$y = \frac{4}{5}x + 4100$

plug in.

$x^2 + \left(\frac{4}{5}x + 4100\right)^2 = 4100^2$

$x^2 + \frac{16}{25}x^2 + 6560x + 4100^2 = 4100^2$

$\frac{41}{25}x^2 + 6560x = 0$

(You could use the quad. formula here.)

$x\left(\frac{41}{25}x + 6560\right) = 0$

$x = 0$? Nah...

$\frac{41}{25}x + 6560 = 0$

$x = \frac{-6560}{41}(25) = -4000$

$y = \frac{4}{5}(-4000) + 4100 = 900$

So:

(0, 4100)

(-4000, 900)

(-5125, 0)

distance inside $= \sqrt{4000^2 + 3200^2} \approx 5122.5$

Time inside $= \frac{\text{dist}}{\text{speed}} = \frac{5122.5}{100} = 51.22$ ms

distance outside $= \sqrt{1125^2 + 900^2} = 1440.7$

Time outside $= \frac{\text{dist}}{\text{speed}} = \frac{1440.7}{200} = 7.20$ ms

Total time $= 51.22 + 7.20$

$= 58.42$

2. Wing and a rabbit are walking around the xy-plane for some reason.

(a) **[5 points]** Wing begins at $(-2, 4)$ and walks at a constant speed in a straight line toward $(5, 6)$, reaching it in 10 seconds.

Give **parametric equations** for Wing's coordinates after t seconds.

$x_0 = -2$ $\quad y_0 = 4$

$x_1 = 5$ $\quad y_1 = 6$

$\Delta x = 7$ $\quad \Delta y = 2$

$\Delta t = 10$

$$x = -2 + \frac{7}{10}t \qquad y = 4 + \frac{2}{10}t$$

(b) **[6 points]** The rabbit begins at $(3.6, -5)$ and hops toward $(-0.9, -2.2)$ with a constant speed of 0.5 units per second.

Give **parametric equations** for the rabbit's coordinates after t seconds.

$x_0 = 3.6$ $\quad y_0 = -5$

$x_1 = -0.9$ $\quad y_1 = -2.2$

$\Delta x = -4.5$ $\quad \Delta y = 2.8$

$\Delta t = 10.6$

How long does it take? The rabbit travels a distance of

$$\sqrt{(-0.9-3.6)^2 + (-2.2-(-5))^2} = 5.3 \text{ units.}$$

$$\Delta t = \frac{\text{dist}}{\text{speed}} = \frac{5.3}{0.5} = 10.6.$$

$$x = 3.6 + \frac{-4.5}{10.6}t \qquad y = -5 + \frac{2.8}{10.6}t$$

(c) **[4 points]** When is Wing **directly north** of the rabbit?

When their x-coordinates are equal!

$$-2 + \frac{7}{10}t = 3.6 + \frac{-4.5}{10.6}t$$

$$\left(\frac{7}{10} + \frac{4.5}{10.6}\right)t = 5.6$$

$$t = \frac{5.6}{\frac{7}{10} + \frac{4.5}{10.6}} \approx 4.98 \text{ seconds}$$

Note: to check that Wing really is north of the rabbit, plug this t into the y-equations from (a) and (b) and make sure Wing's coordinate is greater. It is.

3. Consider the following multipart function:

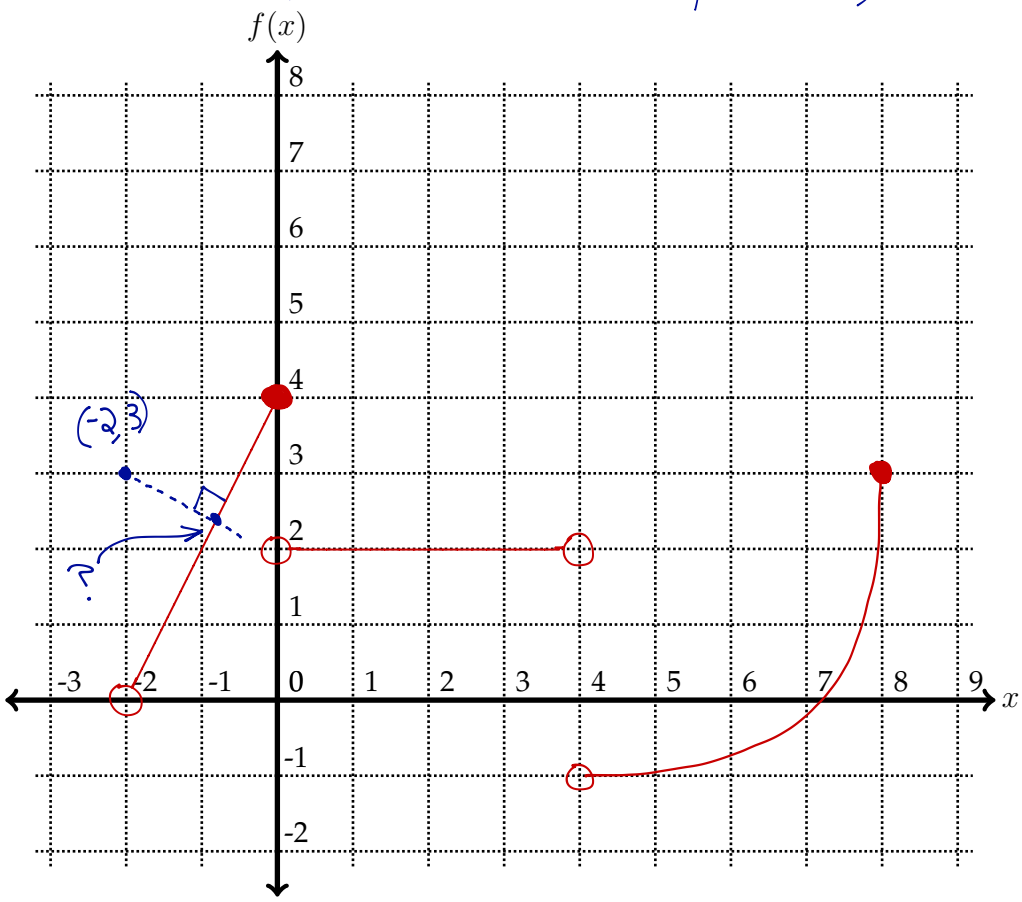
$$f(x) = \begin{cases} 2x+4 & \text{if } -2 < x \le 0 \\ 2 & \text{if } 0 < x < 4 \\ 3 - \sqrt{16-(x-4)^2} & \text{if } 4 < x \le 8 \end{cases}$$

line w/ slope 2, y-intercept 4

horizontal line

lower semicircle w/ center (4,3) radius 4

(a) **[9 points]** Plot $f(x)$ in the grid below. It's okay if you're not a great artist, but please label your graph clearly. (Blue line is for part (b).)



(b) **[6 points]** Find the point on the graph of $f(x)$ which is **closest** to $(-2, 3)$. (Give both coordinates.)

We want the intersection of the line $y = 2x+4$ and the perpendicular (with slope $\frac{-1}{2}$) through $(-2,3)$.

$y = \frac{-1}{2}(x+2)+3$

$2x+4 = \frac{-1}{2}(x+2)+3$

$2.5x = -2$

$x = -0.8$

$y = 2(-0.8)+4 = 2.4$

So $\boxed{(-0.8, 2.4)}$ is the closest point.

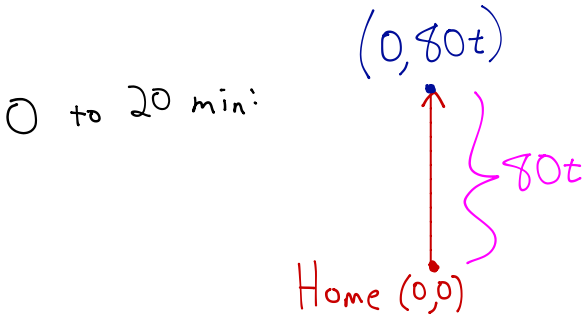
4. **[15 points]** Francie is bored at home when she decides to walk around the neighborhood for a little while and make a math problem.

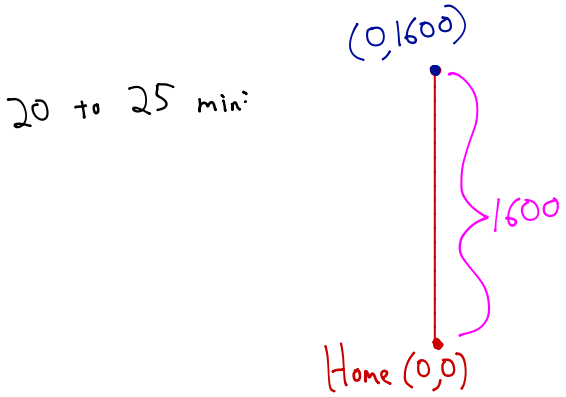
   First, Francie walks north for 20 minutes at a constant speed of 80 meters per minute.

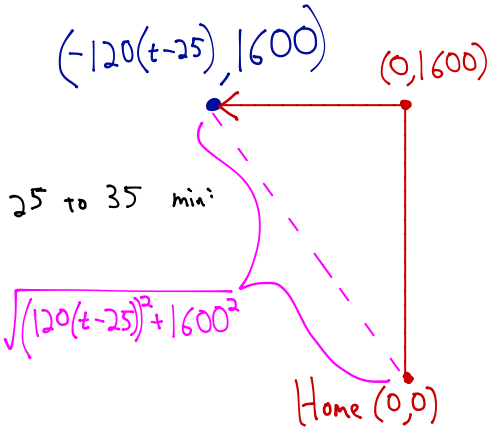
   Then she stands in place for 5 minutes, checking her phone or whatever.

   Then she walks west for 10 minutes at a constant speed of 120 meters per minute.

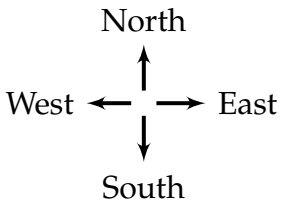
   Write a **multipart function** $f(t)$ for the distance from Francie to her house (in meters), t minutes after she starts walking.







$$f(t) = \begin{cases} 80t & \text{if } 0 \le t \le 20 \\ 1600 & \text{if } 20 \le t \le 25 \\ \sqrt{(120(t-25))^2 + 1600^2} & \text{if } 25 \le t \le 35 \end{cases}$$



*Here's a compass rose, in case you get lost.