Math 120 B,C,D - Autumn 2013
Midterm Exam Number Two Solutions

1. (a) The number of **copies** Jessa sells is a linear function of the price per copy, and using the two points we get that it's given by the equation $n(x) = 230 - 3x$. The **total amount of money** she gets is the number of copies multiplied by the price (x), so we get a solution of $f(x) = x(230 - 3x) = -3x^2 + 230x$.

   (b) The previous solution is a downward-pointing parabola, so we want the x-coordinate of the vertex, which is $x = h = -b/(2a) = -230/(-6) \approx \$38.33$.

2. (a) $f(g(x)) = f(e^x + 3) = \sqrt{e^x + 3} + 2(e^x + 3)$.

   (b) We set $x = \sqrt{y} + 2y$ and attempt to solve for y. First, subtracting y and squaring yields
   $y = x^2 - 4xy + 4y^2$, so $4y^2 + (-4x - 1)y + x^2 = 0$. Using the quadratic formula gives

   $$y = \frac{4x + 1 \pm \sqrt{(-4x-1)^2 - 16x^2}}{8} = \frac{4x + 1 \pm \sqrt{1 - 8x}}{8}$$

   This isn't quite a function yet, because the $\pm$ in the numerator means we have two values of y for every x. We need to determine whether it should be a + or a $-$.
   Now, in the original function, $f(0) = 0$, so in the inverse function, we also want $f^{-1}(0) = 0$. The only way the above function is zero when $x = 0$ is if we're subtracting, so we've got:

   $$f^{-1}(x) = \frac{4x + 1 - \sqrt{1 - 8x}}{8}$$

   (c) $h(x) = 2(\sqrt{x} + 2x) - 3$.

3. (a) Catoma's initial population is 10,000 and it grows by 40% every 8 years, so its population is given by $c(t) = 10000 \cdot (1.4)^{t/8}$.

   (b) Vellebue's population doubles every 15 years, so its population is given by $P \cdot 2^{t/15}$ for some P. From part (a), we can determine that Catoma's population in 2006 was $10000 \cdot (1.4)^{16/8} = 19600$, so in 2006, Vellebue's population was $19600/4 = 4900$. That means $4900 = P \cdot 2^{16/15}$, so $P = 4900/(2^{16/15}) \approx 2339$, which means the population of Vellebue after t years is $v(t) = 2339 \cdot 2^{16/15}$.

   (c) We solve $150000 = 2339 \cdot 2^{t/15}$ by taking the log of both sides and simplifying to get $t \approx 90$ years after 1990, so the population will reach 150000 in 2080.

4. (a) We're looking for a function of the form $f(x) = \dfrac{ax + b}{x + d}$. We know that $f(0) = 40$, $f(100) = 85$, and $a = 130$. Combining the first and third equations tell us that $40 = b/d$, so $b = 40d$. The second equation tells us that $85 = (13000 + b)/(100 + d)$, so $8500 + 85d = 13000 + 40d$, so $d = 4500/45 = 100$, and $b = 4000$.
   So $f(x) = \dfrac{130x + 4000}{x + 100}$.

   (b) In 2050, the population will be $f(150) = 94$ million.