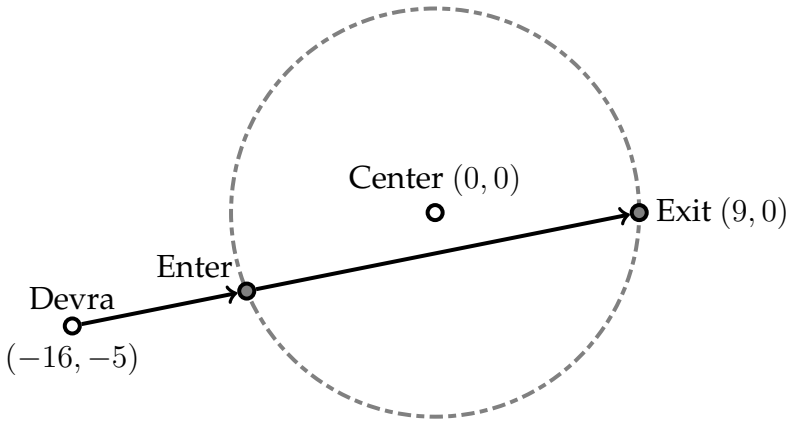
Math 120 B,C,D - Autumn 2013
Midterm Exam Number One Solutions

Version 1, in which Devra's swimming pool has a radius of 9 meters:

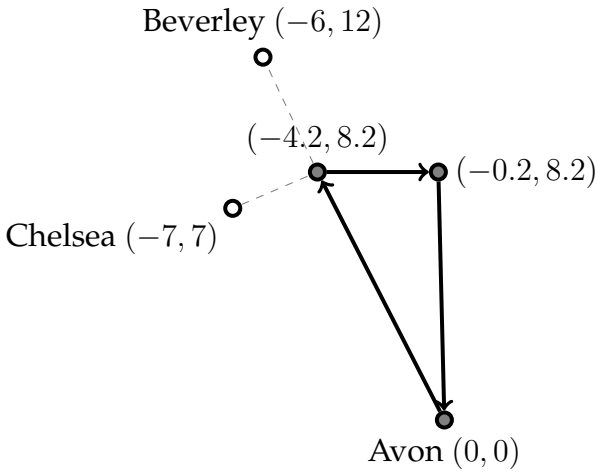
1. Here's a picture. The pool is the gray circle, and we want the time Devra takes to swim from where she enters the pool to where she exits it.

   She walks along the line $y = \frac{1}{5}(x-9)$ until she intersects the circle $x^2+y^2 = 9^2$ at the point $(-108/13, -45/13)$. Using the distance formula we can see how far this is from $(9,0)$, and we divide by her speed to get a time of $45/\sqrt{26} \approx 8.825$ seconds.



2. Avon walks towards Beverley along the line $y = -2x$, and stops when that line intersects the perpendicular line through Chelsea. The equation of that second line is $y = \frac{1}{2}(x+7)+7$, and the two lines intersect at $(-4.2, 8.2)$. Four feet east of that is $(-0.2, 8.2)$, and we want to sum the distances between those three points. The total distance is:

$$d = \sqrt{4.2^2 + 8.4^2} + 4 + \sqrt{0.2^2 + 8.4^2} \approx 21.794$$



3. (a) Iphigenia starts at $(6, 2)$, her y-coordinate is constant, and her x-coordinate decreases by 4 units every second, so her equations are $x = 6 - 4t$, $y = 2$.

   (b) Ethelbert starts at $(0, 0)$ and walks towards $(6, 2)$ in 3 seconds, so $\Delta x = 6$, $\Delta y = 2$, $\Delta t = 3$, and we get $x = 0 + \frac{6}{3}t$, $y = 0 + \frac{2}{3}t$.

   (c) Plug their parametric equations into the distance formula. The distance is

$$d = \sqrt{[(6 - 4t) - (2t)]^2 + \left[2 - (\frac{2}{3}t)\right]^2}$$

4. (a) We've got part of a semicircle, and then two line segments:

$$f(x) = \begin{cases} 2 + \sqrt{3^2 - (x+4)^2} & \text{if } -4 \le x \le -1 \\ -\frac{1}{2}(x-1) + 1 & \text{if } -1 \le x \le 1 \\ \frac{5}{3}(x-1) + 1 & \text{if } 1 \le x \le 4 \end{cases}$$

   (b) To solve $f(x) = 4$, we set 4 equal to each of the three pieces from part (a). (It's okay to say that it obviously doesn't intersect the middle line segment, if you look at the picture.)

   Solving $4 = 2 + \sqrt{3^2 - (x+4)^2}$ yields $x = -4 \pm \sqrt{5}$, but $-4 - \sqrt{5}$ is outside the domain, so we get $x = -4 + \sqrt{5} \approx -1.764$ as one solution.

   Solving $4 = (5/3)(x - 1) + 1$ yields $x = 2.8$. So $x = 2.8$ or $x = -4 + \sqrt{5}$.