Math 120 - Autumn 2013
Final Exam
December 7, 2013

Name: ____________________________ Student ID no. : ________________

Signature: ________________________________ Section: ____________

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
Total	80	

- Complete all eight questions.
- Show all work for full credit.
- You may use a scientific calculator during this examination. Graphing calculators are not allowed. Also, other electronic devices are not allowed, and should be turned off and put away for the duration of the exam.
- If you use a trial-and-error or guess-and-check method when an algebraic method is available, you will not receive full credit.
- You may use one hand-written 8.5 by 11 inch page of notes. Write your name on your notesheet and turn it in with your exam.
- You have 170 minutes to complete the exam.

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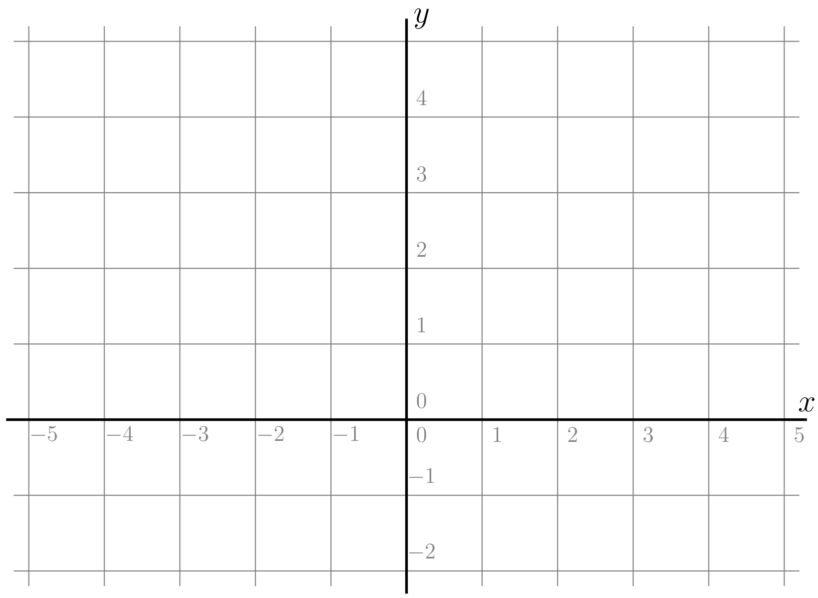
1. $f(x)$ is the multipart function whose rule is given below.

$$f(x) = \begin{cases} 2 + \sqrt{4 - (x+4)^2} & \text{if } -4 \le x < -2 \\ \frac{1}{2}x + 3 & \text{if } -2 \le x \le 2 \\ -2x + 8 & \text{if } 2 < x \le 4 \end{cases}$$

(a) Sketch the graph of $f(x)$.



(b) What point on the graph of $f(x)$ is **closest** to the point (2,2)?

2. Sage is deciding how to price the goat cheese dumplings at her new restaurant. After some investigation, she has decided that the number of bowls of dumplings she'll sell each day is a **linear** function of the price.

   If she prices the dumplings at \$4.00 per bowl, she'll sell 60 bowls per day.

   If she prices the dumplings at \$7.50 per bowl, she'll sell 32 bowls per day.

   (a) Give a linear function $d(x)$ for the **number of bowls of dumplings** sold each day, if Sage prices each one at $\$x$.

   (b) Give a function for $p(x)$ for the **total amount of money earned** each day, if Sage prices each dumpling at $\$x$.

   (c) In order to maximize the total amount of money earned each day, **what price** should Sage charge?

3. A bag of popcorn is popping in the microwave. The number of un-popped kernels is a linear-to-linear rational function of time.

   After 1 minutes, there are 150 un-popped kernels left in the bag.

   After 2 minutes, there are 128 un-popped kernels left in the bag.

   After 5 minutes, there are 95 un-popped kernels left in the bag.

   (a) Give a **linear-to-linear** rational function $f(t)$ to express the number of un-popped kernels after t minutes.

   (b) **Do not attempt this problem at home.**

   If the bag is left in the microwave indefinitely, how many un-popped kernels will there be in the long run, according to your function from part (a)?

4. A seriously tall dude is standing off in the distance. Really, he's very very tall.

   You measure the angle of elevation from the ground to the top of his head. It's $30°$.

   After walking 20 feet towards him along the flat ground, you measure the angle of elevation again. This time it's $36°$.

   (a) **How tall** is this seriously tall dude?

   (b) After the second measurement, he puts on an outrageously large hat.

   You measure the angle of elevation once again from your new location, this time including the hat as well. The angle is $40°$.

   How tall is the **hat**?

5. Alexandra and Boris are running around a circular track. The track has a radius of 50 meters.

   Boris starts from the easternmost point of the track, and runs clockwise.

   Alexandra starts at the same time from the northernmost point of the track, and runs counter-clockwise.

   Boris runs at a constant speed of 4 meters per second, and passes Alexandra for the first time after 20 seconds.

   (a) What is Alexandra's speed in meters per second?

   (b) After running for 1000 seconds, who is farther south, Boris or Alexandra? Show all work.

6. Ping has been measuring her level of joy on a scale from 0 (no joy) to 100 (maximum joy). She has found that her level of joy can be expressed as a sinusoidal function. Today, her joy measurement is at its lowest level: 0.

   The next time her joy measurement will be at its maximum is 300 days from now.

   (a) Write the sinusoidal function for Ping's joy level t days after today.

   (b) Over the next 1000 days, **how much of the time** will her joy be above 70?

7. An alpaca and a bison are travelling along straight lines, at constant speeds, in the xy-plane. They start moving at the same time. The units of the xy-plane are kilometers (e.g., the point $(0, 1)$ is one kilometer away from $(0, 0)$).

   The alpaca starts from the point $(2, 10)$, and heads directly toward the point $(20, -3)$, reaching it after 4 hours.

   The bison starts from the point $(-5, -5)$ and heads directly toward the point $(4, 7)$. The bison moves at a constant speed of 6 km/hr.

   (a) Write parametric equations for the alpaca's position t seconds after it starts walking.

   (b) Write parametric equations for the bison's position t seconds after it starts walking.

   (c) Does the alpaca or the bison cross the x-axis first? Show your work.

8. (a) Let $f(x) = 3x - 1$ and $g(x) = 2x + 6$. Let $h(x) = f(g(x))$. Solve the equation $h(x) = 0$.

(b) Let $f(x) = 3x - 1$. Let $j(x)$ be the function whose graph is that of $f(x)$ shifted 3 units to the left and 4 units upward. Find $j^{-1}(x)$.

(c) Let $f(x) = 3x - 1$ and $g(x) = 2x + 6$. Solve for x: $2^{f(x)} = 3^{g(x)}$.