Math 120 Autumn 2012
Final Exam
December 8, 2012

Name: ______________________ Student ID no. : ______________

Signature: ______________________ Section: ______________

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
Total	80	

- Complete all eight questions.
- Show all work for full credit.
- You may use a scientific calculator during this examination. Graphing calculators are not allowed. Also, other electronic devices are not allowed, and should be turned off and put away for the duration of the exam.
- If you use a trial-and-error or guess-and-check method when an algebraic method is available, you will not receive full credit.
- You may use one hand-written 8.5 by 11 inch page of notes.
- You have 170 minutes to complete the exam.
- You may round your final answers to 2 digits. Do not round before your final answer.

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1. (a) In 2010, the US federal budget was 3.6 trillion dollars. In 2013, it is 3.8 trillion dollars. Compute a linear model for the US budget $B(x)$, in trillions of dollars, x **years after 2005**. Use at least 4 decimal places in all values.

(b) In 2005, the US GDP (gross domestic product) was 12.56 trillion dollars. In 2010, it was 14.45 trillion dollars. Compute a linear model for $G(x)$, the US GDP, in trillions of dollars, x **years after 2005**.

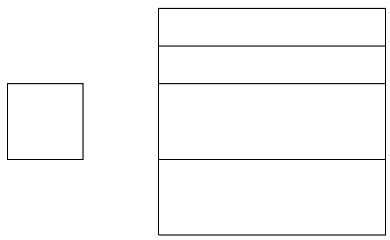
(c) The percentage of the US budget relative to the GDP is given by $P(x) = \dfrac{B(x)}{G(x)} \times 100\%$. Use your answers above to write a linear-to-linear rational function for the percentage $P(x)$, x years after 2005. Find the value that $P(x)$ approaches long-term.

2. In 2005, the gross US federal debt was 7.9 trillion dollars. In 2010, it was 13.5 trillion dollars.

   In 2005, the US GDP (gross domestic product) was 12.56 trillion dollars. In 2010, it was 14.45 trillion dollars.

   Assuming both the federal debt and the GDP grow exponentially, when will the US debt equal twice the GDP? Give your answer in years after 2005.

3. You have 280 meters of fencing with which to make two enclosures. Both will be squares, and one will have three partitions (which separate the square into four spaces). The enclosures might look like this:



   (a) What should the dimensions of the squares be to *minimize* the sum of the areas of the squares?

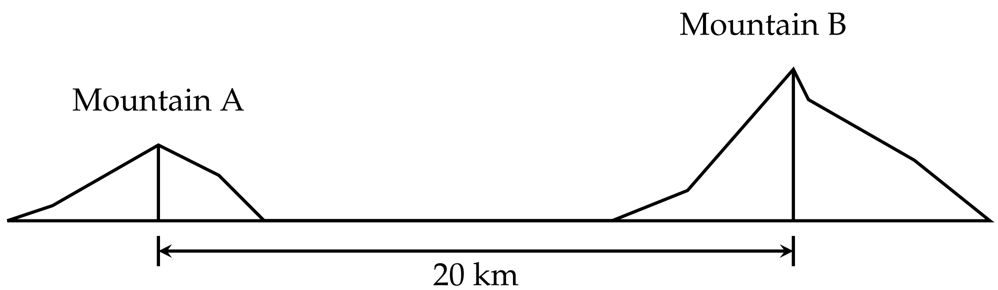
   (b) What should the dimensions of the squares be to *maximize* the sum of the areas of the squares? Be sure to show all work.

4. You are on a road connecting the bases of Mountain A and Mountain B.

   You look at Mountain A and measure the angle of elevation to the top of Mountain A to be $15°$.

   You then travel 2 km toward Mountain B.

   You measure Mountain B's angle of elevation from your new location to be $17°$.



   Mountain A and Mountain B are 20 km apart as shown in the figure, and Mountain B is exactly twice as tall as Mountain A.

   What is the height of Mountain A?

5. The diameter of a certain cloud in the sky above Seattle is a sinusoidal function of time.

   At 7 AM this morning, the diameter of the cloud was at its minimum, 20 meters.

   The cloud then expanded, and reached its maximum diameter of 26 meters at 11:30 AM this morning.

   From 3 AM this morning to 3 PM this afternoon, for how much time was the cloud's diameter less than 21.5 meters?

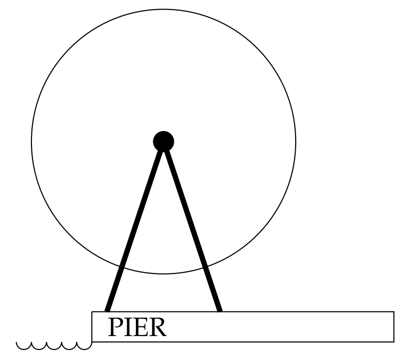
6. Let $f(x) = \frac{1}{3}|x-1| + 8$.

   (a) A *fixed point* of a function $g(x)$ is a number k such that $g(k) = k$. Find all fixed points of $f(x)$. Show all work.

   (b) Let $h(x)$ be the function we get by restricting $f(x)$ to the domain $x \leq 1$. Find $h^{-1}(x)$ and specify its domain.

7. The Seattle Great Wheel opened in June 2012. It is the largest observation wheel on the west coast. It has a diameter of 175 feet, and a height of 200 feet above the pier. Rides last 12 minutes and include three rotations.

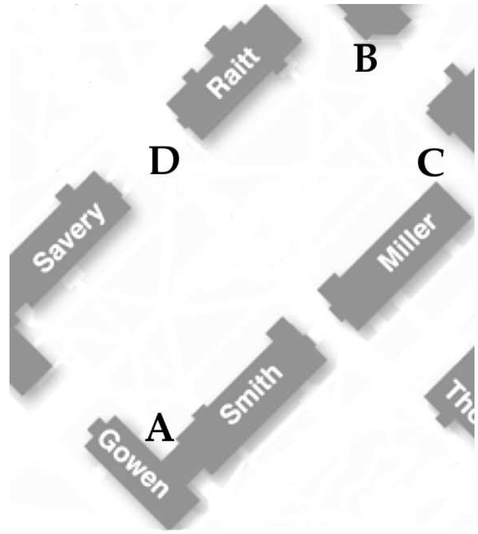
   (a) What is the angular speed (in radians per minute) and the linear speed (in feet per minute) of a rider on this wheel?



   (b) Suppose you start your ride at the bottom of the wheel. Write your height above the pier, in feet, as a function of the number of minutes, t, since the beginning of your ride.

   (c) Suppose the wheel is at rest, and the owner wants to stretch a horizontal string of holiday lights across the wheel, at a height of 180 feet above the pier. How long a string does he need?

8. At noon, Alex exits Smith Hall at point A on the map shown and starts walking at constant speed directly towards the Art building (point B), hoping for a cup of coffee at Parnassus. She gets to point B after 60 seconds.



At the same time (noon), Matt is at point C (near the Music building), walking straight towards point D at a uniform speed of 2 feet per second, rushing to his next class.

Point B is 60 feet east and 110 feet north of point A. Point D is 80 feet due north of point A, and point C is 70 feet due east of point D.

Impose a coordinate system with the origin at point A.

(a) Determine parametric equations for Alex's coordinates t seconds past noon.

(b) Determine parametric equations for Matt's coordinates t seconds past noon.

(c) What is the closest distance between Matt and Alex during their treks across the Quad?