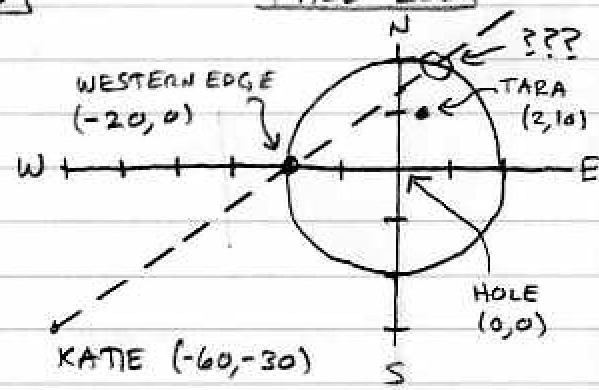
MATH 120 — EXAM 1 SOLUTIONS — FALL 2009



① (a) CIRCLE: $x^2+y^2=20^2$

LINE: $m=\frac{0--30}{-20--60}=\frac{30}{40}=\frac{3}{4}$

$y=m(x-x_1)+y_1=\frac{3}{4}(x--20)+0$

$y=\frac{3}{4}x+15=0.75x+15$

INTERSECTION: $x^2+\left(\frac{3}{4}x+15\right)^2=20^2$

$x^2+\frac{9}{16}x^2+\frac{90}{4}x+225=400$

$\frac{25}{16}x^2+\frac{45}{2}x-175=0 \Leftrightarrow 1.5625x^2+22.5x-175=0$

QUAD. FORMULA: $x=\frac{-22.5\pm\sqrt{22.5^2-4(1.5625)(-175)}}{2(1.5625)}$

$x=\frac{-22.5\pm\sqrt{1600}}{3.125}=\frac{-22.5\pm 40}{3.125}$

$x=\frac{-62.5}{3.125}=-20$ ✓ OR $x=\frac{17.5}{3.125}=5.6=\frac{28}{5}$

BACK SUBSTITUTION: $y=\frac{3}{4}x+15=\frac{3}{4}(5.6)+15=19.2=\frac{96}{5}$

NORTH EAST

$$\boxed{(x,y)=(5.6,\ 19.2)=\left(\frac{28}{5},\frac{96}{5}\right)}$$

(b) PERPENDICULAR THROUGH TARA: $m=-\frac{1}{(3/4)}=-\frac{4}{3}=-1.\overline{3}$

$y=m(x-x_1)+y_1$ ← POINT: (2, 10)

$y=-\frac{4}{3}(x-2)+10=-\frac{4}{3}x+\frac{8}{3}+10$

$y=-\frac{4}{3}x+\frac{38}{3}=-1.\overline{3}x+12.\overline{6}$

INTERSECTION: $\frac{3}{4}x+15=-1.\overline{3}x+12.\overline{6}$

$2.08\overline{3}x=-2.\overline{3}$

$x=\frac{-2.\overline{3}}{2.08\overline{3}}=-1.12$

BACK SUB: $y=\frac{3}{4}x+15=\frac{3}{4}(-1.12)+15=14.16$

DISTANCE: $\sqrt{(2--1.12)^2+(10-14.16)^2}$

$=\sqrt{9.7344+17.3056}=\sqrt{27.04}$

$=\boxed{5.2 \text{ feet}}$

② (a) $x = a + bt$ $\quad y = c + dt$

$t = 0,\ x = -2 \Rightarrow a = -2$ $\quad t = 0,\ y = 11 \Rightarrow c = 11$

Peyton's time to second point $= \frac{\text{DIST}}{\text{SPEED}} = \frac{\sqrt{(-13-11)^2 + (5-(-2))^2}\ \text{ft}}{4\ \text{ft/sec}}$

$= \frac{\sqrt{625}}{4} = \frac{25}{4} = 6.25$ seconds

$t = 6.25,\ x = 5 \Rightarrow 5 = -2 + b(6.25) \Rightarrow b = \frac{7}{6.25} = 1.12 = \frac{28}{25}$

$t = 6.25,\ y = -13 \Rightarrow -13 = 11 + d(6.25) \Rightarrow d = \frac{-24}{6.25} = -3.84 = -\frac{96}{25}$

$$\boxed{x = -2 + 1.12t \qquad y = 11 - 3.84t}$$

(b) Peyton crosses y-axis when $x = 0$

From part (a), this implies $0 = -2 + 1.12t \Rightarrow t = \frac{2}{1.12} = \frac{25}{14}$

$t \approx 1.785714286$ sec

The y-coord at this time is $y = 11 - 3.84t = 4.\overline{142857} = \frac{29}{7}$

Peyton crosses the y-axis at $(0,\ 4.\overline{142857})$

Eli's speed $= \frac{\text{DIST}}{\text{TIME}} = \frac{\sqrt{(10-0)^2 + (7-4.142857)^2}}{1.78571426}\ \frac{\text{ft}}{\text{sec}}$

$\approx \frac{\sqrt{108.1632653}}{1.785714286} \approx \boxed{5.82406791\ \text{ft/sec}}$

$\boxed{\text{Alternate sol'n for (b)}}$

Peyton's Path: $m = \frac{-13-11}{5--2} = -\frac{24}{7} \Rightarrow y = -\frac{24}{7}(x - -2) + 11$

Crosses y-axis: $x = 0 \Rightarrow y = -\frac{48}{7} + 11 = -\frac{48}{7} + \frac{77}{7} = \frac{29}{7} = 4.\overline{142857}$

Peyton's Time: $\text{TIME} = \text{DIST}/\text{SPEED} = \sqrt{(0--2)^2 + (4.\overline{142857} - 11)^2}\,/\,4$

$= 1.785714286$

Then complete as above.

③ (a) $\boxed{\begin{array}{l} x = -\dfrac{b}{2a} = -\dfrac{53}{2(-2)} = 13.25 \\ y = -2(13.25)^2 + 53(13.25) = 351.125 \end{array}}$

(b) Want to know x when $y = 100$.

$$100 = -2x^2 + 53x$$

$$0 = -2x^2 + 53x - 100$$

QUAD FORMULA: $x = \dfrac{-53 \pm \sqrt{53^2 - 4(-2)(-100)}}{2(-2)}$

$$x = \frac{-53 \pm \sqrt{2009}}{-4}$$

$$x \approx \frac{-8.178130338}{-4} \quad \text{OR} \quad x \approx \frac{-97.82186966}{-4}$$

$$x \approx 2.044532584 \quad \text{OR} \quad x = 24.45546742$$

The edge of the silo is at $x = 25$, so the ball **DOES** hit the "top" at the point $\boxed{(24.46, 100)}$

(c) $$\frac{f(x+h) - f(x)}{h} = \frac{[-2(x+h)^2 + 53(x+h)] - [-2x^2 + 53x]}{h}$$

$$= \frac{-2(x^2 + 2xh + h^2) + \cancel{53x} + 53h + 2x^2 - \cancel{53x}}{h}$$

$$= \frac{-\cancel{2x^2} - 4xh - 2h^2 + 53h + \cancel{2x^2}}{h}$$

$$= \frac{-4xh}{h} - \frac{2h^2}{h} + \frac{53h}{h}$$

$$= \boxed{-4x - 2h + 53}$$

Alternate sol'n for (b)

$f(25) = -2(25)^2 + 53(25) = 75 \Rightarrow$ The ball must pass a height of 100 before $x = 25$. (so it lands on top) Then you still would need to solve for $y = 100$ to get the location.

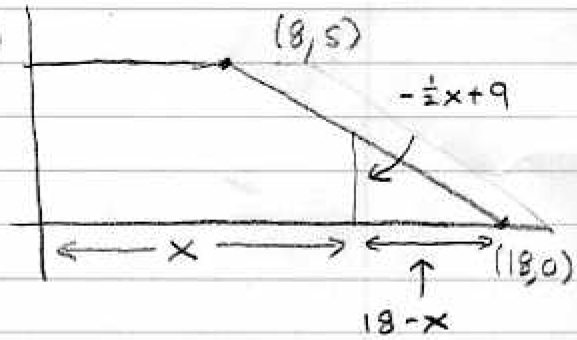
(4) (a) Horizontal Line: $\boxed{y=5}$

Decreasing Line: $m = \frac{0-5}{18-8} = \frac{-5}{10} = -\frac{1}{2}$

$y = m(x-x_1)+y_1 = -\frac{1}{2}(x-18)+0$

$\boxed{y = -\frac{1}{2}x+9}$

$$\boxed{y = \begin{cases} 5, & \text{if } 0 \le x \le 8; \\ -\frac{1}{2}x+9, & \text{if } 8 \le x \le 18. \end{cases}}$$



(b) For $0 \le x \le 8$, AREA OF RECTANGLE = (WIDTH)(HEIGHT) = $(x)(5)$
$= 5x$

For $8 \le x \le 18$, $\underbrace{\text{AREA OF WHOLE}}$ − AREA OF SMALL TRIANGLE

$[8 \cdot 5 + \frac{1}{2}(10)(5)] - [\frac{1}{2}(18-x)(-\frac{1}{2}x+9)]$

ALL ACCEPTABLE

$= 65 - \frac{1}{2}(18-x)(-\frac{1}{2}x+9)$

$= 65 - \frac{1}{4}(18-x)(-x+18)$ ← $(18-x)^2$

$= 65 - \frac{1}{4}(18-x)^2 = 65 - \frac{1}{4}(324-36x+x^2)$

$= 65 - 81 + 9x - \frac{1}{4}x^2 = -16+9x-\frac{1}{4}x^2$

$$\boxed{\text{AREA} = \begin{cases} 5x, & \text{if } 0 \le x \le 8; \\ -16+9x-\frac{1}{4}x^2, & \text{if } 8 \le x \le 18. \end{cases}}$$

(c) HALF OF WHOLE $= \frac{1}{2}(65) = 32.5$

THIS AREA OCCURS IN THE FIRST REGION

BECAUSE IT IS LESS THAN 40 (= AREA OF LARGE RECTANGLE).

THUS, $5x = 32.5 \Rightarrow \boxed{x = 6.5}$