MATH 120 FALL 2008 EXAM 2

VERSION 1

1 $y = \frac{ax+b}{x+d}$

(i) horizontal asymptote at $y = 8 \Rightarrow \boxed{a = 8}$

(ii) $5 = \frac{8(10)+b}{(10)+d} \Rightarrow 50 + 5d = 80 + b \Rightarrow \boxed{b = 5d - 30}$

(iii) $6 = \frac{8(20)+b}{(20)+d} \Rightarrow 120 + 6d = 160 + b \Rightarrow \boxed{b = 6d - 40}$

Combining (ii) and (iii) gives $5d - 30 = 6d - 40$

Thus, $\boxed{10 = d}$

$\Rightarrow \boxed{b = 5d - 30 = 20}$

$y = \frac{8x+20}{x+10}$

$7 = \frac{8x+20}{x+10} \Rightarrow 7x + 70 = 8x + 20$

$\Rightarrow \boxed{x = 50 \text{ days}}$

2 (a) $f(f(x)) = a(ax+5)+5 = a^2x + 5a + 5 \overset{??}{=} 4x - 5$

$\Rightarrow a^2 = 4$ AND $5a + 5 = -5$

$a = \pm 2$ AND $a = -2$

ONLY ANSWER $\Rightarrow \boxed{a = -2}$

(b) $y = x^2 - 4x \quad x \leq 2$

$\Rightarrow 0 = x^2 - 4x - y$

$\Rightarrow x = \frac{4 \pm \sqrt{16 - 4(-y)}}{2} =$

So $\boxed{f^{-1}(y) = \frac{4 - \sqrt{16 + 4y}}{2} = 2 - \sqrt{4 + y}}$

(c) $\tan(35^\circ) = \frac{40+A}{100+B} \Rightarrow (100+B)\tan(35^\circ) = 40 + A$

$\tan(55^\circ) = \frac{A}{B} \Rightarrow B\tan(55^\circ) = A$

Combining $(100+B)\tan(35^\circ) = 40 + B\tan(55^\circ)$

$\Rightarrow 100\tan(35^\circ) + B\tan(35^\circ) = 40 + B\tan(55^\circ)$

$\Rightarrow 100\tan(35^\circ) - 40 = B(\tan(55^\circ) - \tan(35^\circ))$

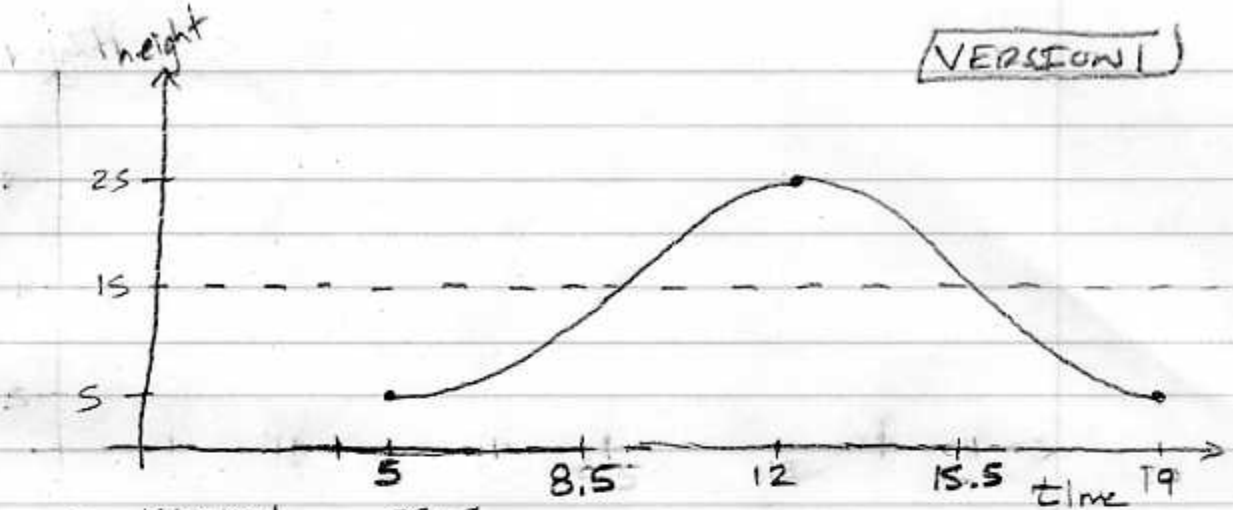
$\Rightarrow B = \frac{100\tan(35^\circ) - 40}{\tan(55^\circ) - \tan(35^\circ)} \approx 41.240672$

$A = B\tan(55^\circ) \approx 58.897783$

$\boxed{A \approx 59 \text{ ft} \quad B \approx 41 \text{ ft}}$

VERSION I

3 (a)



$A = \frac{MAX - MIN}{2} = \frac{25-5}{2} = 10$

$D = \frac{MAX + MIN}{2} = \frac{25+5}{2} = 15$

$B = 14$

$C = 8.5 \quad (= 12 - \frac{14}{4})$

← or any number of the form $8.5 + 14k$, $k \in \mathbb{Z}$.

$$\boxed{h(t) = 10 \sin\left[\frac{2\pi}{14}(t - 8.5)\right] + 15}$$

(b) $h(0) = 10 \sin\left[\frac{2\pi}{14}(0 - 8.5)\right] + 15$

$\approx \boxed{21.23489802 \text{ inches}}$

$\boxed{21.23 \text{ inches}}$

if you are in degree mode you get the incorrect answer 14.335

4 $\omega = \frac{1 \text{ rev}}{80 \text{ sec}} = \frac{2\pi \text{ rad}}{80 \text{ sec}} = \frac{\pi}{40} \frac{\text{rad}}{\text{sec}}$

$v = 15 \frac{\text{ft}}{\text{sec}}$

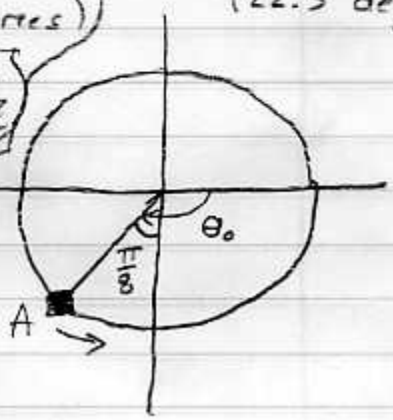
Thus, $v = \omega r \Rightarrow r = \frac{v}{\omega} = \frac{15}{(\pi/40)} = \frac{600}{\pi} \approx 190.9859$ ft

In 5 seconds, angle swept out $= \omega t = \frac{\pi}{40} 5 = \frac{\pi}{8}$ radians (22.5 degrees)

$\theta_0 = -\left(\frac{\pi}{2} + \frac{\pi}{8}\right)$ radians $\quad (-202.5 \text{ degrees})$

$= -\frac{5\pi}{8}$ radians ← or any value $-\frac{5\pi}{8} + 2\pi k$, $k \in \mathbb{Z}$

(a popular other choice is $\frac{11\pi}{8}$)



$\omega = \frac{\pi}{40} \frac{\text{rad}}{\text{sec}}$, $t = 2 \text{ min} = 120 \text{ sec}$

$x = r \cos(\theta_0 + \omega t)$

$\approx 190.9859 \cos\left(-\frac{5\pi}{8} + \frac{\pi}{40} \cdot 120\right)$

$\simeq 73.087$

$y = r \sin(\theta_0 + \omega t) \approx 176.447$

$\boxed{(x, y) = (73.09, 176.45)}$