version 1

MATH 120 $\quad$ EXAM 1 SOL'NS $\quad$ FALL 2008

1 $f(x) = m(x - x_1) + y_1$ = rent in Funtown

$(1960, 100)$ $\quad$ $(1990, 400)$ $\quad$ so $m = \frac{400-100}{1990-1960} = \frac{300}{30} = 10$

$f(x) = 10(x - 1960) + 100 = 10x - 19500$

$g(x) = m(x - x_1) + y_1$ = rent in Boreville

$(1960, 150)$ $\quad$ $(1990, 210)$ $\quad$ so $m = \frac{210-150}{1990-1960} = \frac{60}{30} = 2$

$g(x) = 2(x - 1960) + 150 = 2x - 3770$

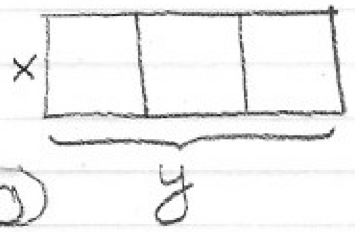
Want $\quad$ Funtown rent = Boreville + 500

$$10x - 19500 = 2x - 3770 + 500$$
$$10x - 19500 = 2x - 3270$$
$$8x - 19500 = -3270$$
$$8x = 16230$$

$$\boxed{x = \frac{16230}{8} = \frac{8115}{4} = 2028.75}$$

2

AREA = xy



FENCING = $4x + 2y = 950$

so $\quad 2y = 950 - 4x$

$\Rightarrow y = 475 - 2x$

Thus,

AREA $= x(475 - 2x) = \underbrace{-2}_{a}x^2 + \underbrace{475}_{b}x$ $\quad c = 0$

$a < 0 \Rightarrow$ downward facing parabola so the vertex is a maximum.

VERTEX : $\quad x = -\frac{b}{2a} = -\frac{475}{2(-2)} = 118.75$

$$\boxed{x = 118.75 \text{ ft}}$$

$$y = 475 - 2x = 475 - 2(118.75) = 237.5$$

$$\boxed{y = 237.5 \text{ ft}}$$

version 1

3 (a) $f(2) = (2)^2 + 4(2) = 12$

$f(2-h) = (2-h)^2 + 4(2-h) = 4 - 4h + h^2 + 8 - 4h$

$\Rightarrow f(2-h) = 12 - 8h + h^2$

$f(h) = h^2 + 4h$

So

$$\frac{f(2) - f(2-h) + 2f(h)}{h} = \frac{(12) - (12 - 8h + h^2) + 2(h^2 + 4h)}{h}$$

$$= \frac{\cancel{12} - \cancel{12} + 8h - h^2 + 2h^2 + 8h}{h}$$

$$= \frac{16h + h^2}{h} = \boxed{16 + h}$$

(b) Completing the square for $f(x)$ gives

$f(x) = x^2 + 4x = x^2 + 4x + 4 - 4$

$f(x) = (x+2)^2 - 4$ ← y-coord of vertex $= -4$

x-coord. of vertex $= -2$

or we can use the formulas

$$\boxed{\begin{array}{l} x = -\frac{b}{2a} = -\frac{4}{2 \cdot 1} = -2 \\ y = (-2)^2 + 4(-2) = -4 \end{array}}$$

$c - ah^2 = -4$ works as well

x-coord. of vertex of $g(x)$ = x-coord of vertex of $f(x)$

$-\frac{b}{2(10)} = -2$

$\Rightarrow -b = -40$

$\Rightarrow \boxed{b = 40}$

version 1

4 circle $= x^2+y^2=10^2$

horiz. line: $y=2$

turning point: $10 \frac{mi}{\cancel{hr}} \cdot \frac{1}{2} \cancel{hr} = 5$ miles

$-3+5=2$

$(2,2)$

vertical line: $x=2$

intersection: $2^2+y^2=10^2 \Rightarrow y^2=96$

$\Rightarrow y=\pm\sqrt{96}$

$x=2$

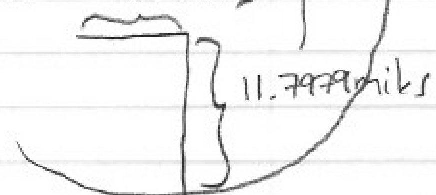
$y=-\sqrt{96} \approx -9.7979589711 3$

$$\text{TIME} = \frac{\text{DIST}}{\text{SPEED}} = \frac{5 \text{ miles} + (2 - -9.7979589711 3) \text{ miles}}{10 \text{ mi/hr}}$$

$= (7+\sqrt{96})/10$ hours

≈ 1.679795897 hrs

$\boxed{1.68 \text{ hrs}}$



5 (a) line 1: $y=m(x-x_1)+y_1$

$(0,0) \quad (2,6) \qquad m=\frac{6-0}{2-0}=3$

$y=3(x-0)+0 \qquad y=3x$ if $0\le x\le 2$

line 2: $y=m(x-x_1)+y_1$

$(2,6) \quad (5,0) \qquad m=\frac{6-0}{2-5}=-2$

$y=-2(x-5)+0=-2x+10$ if $2\le x\le 5$

so
$$\boxed{y=\begin{cases} 3x, & \text{if } 0\le x\le 2;\\ -2x+10, & \text{if } 2\le x\le 5.\end{cases}}$$

(b) region 1: AREA $=\frac{1}{2}xy$

and $y=3x$

So AREA $=\frac{3}{2}x^2$ if $0\le x\le 2$

region 2: AREA = WHOLE − SMALL TRIANGLE

$=15-\frac{1}{2}(5-x)y$

and $=15-\frac{1}{2}(5-x)(-2x+10)=$

$$\boxed{\text{AREA}=\begin{cases} \frac{3}{2}x^2 & \text{if } 0\le x\le 2;\\ 15-\frac{1}{2}(5-x)(-2x+10) & \text{if } 2\le x\le 5,\end{cases}}$$

$=-x^2+10x-10$