1. (10 points) Maggie is a high jumper. The more hours that she practices, the higher her average jump height will be. If she practices for no hours, her average jump height is 3 feet. If she practices for 30 hours, her average jump height is 5 feet. As she practices more and more, her average jump height approaches (but never exceeds) 7 feet.

(a) Find the linear-to-linear rational model that gives Maggie's average jump height, y, in terms of the number of hours of practice, x.

$$y = \frac{ax+b}{x+d}$$

① $x=0 \Rightarrow y=3$

② $x=30 \Rightarrow y=5$

③ horiz. asymptote $y=7$

③ $\Rightarrow \boxed{a=7}$ $\quad y=\frac{7x+b}{x+d}$

① $\Rightarrow 3 = \frac{7(0)+b}{(0)+d} \Rightarrow \boxed{b=3d}$

② $\Rightarrow 5 = \frac{7(30)+b}{(30)+d} \Rightarrow 5(30+d) = 210+b \Rightarrow \boxed{150+5d = 210+b}$

combining $\Rightarrow$ $5d = 60+3d \Rightarrow 2d=60 \Rightarrow \boxed{d=30}$

and $\boxed{b=3d=90}$

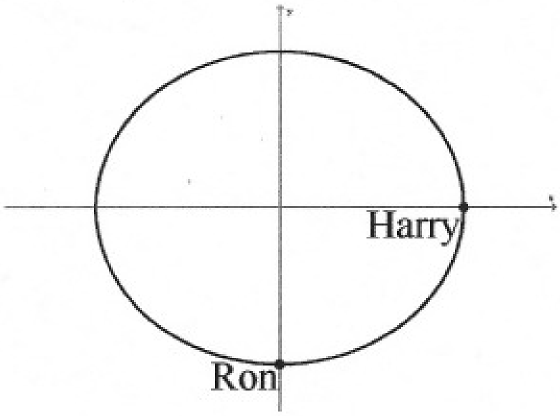
$$\boxed{y = \frac{7x+90}{x+30}}$$

(b) If Maggie wants to have an average jump height of 6 feet, how many hours should she practice?

$$6 = \frac{7x+90}{x+30} \Rightarrow 6x+180 = 7x+90$$

$$\Rightarrow \boxed{x = 90 \text{ hours}}$$

2. (10 points) Ron and Harry are both running counterclockwise on a circular track with radius 10 feet. Ron starts at the southernmost point and Harry is the easternmost point. Ron is running at 2 feet/sec and Harry completes one lap in 30 seconds.



(a) Give Harry's x and y coordinates after 3 seconds.

$\omega = \frac{1 \text{ rev}}{30 \text{ sec}} = \frac{2\pi \text{ rad}}{30 \text{ sec}} = \frac{\pi}{15} \text{ rad/sec}$

$\theta_0 = 0 \text{ rad}$

$\theta = \omega t + \theta_0 = \frac{\pi}{15} \frac{\text{rad}}{\cancel{\text{sec}}} 3 \cancel{\text{sec}} + 0 \text{ rad}$

$\theta = \frac{\pi}{5} \text{ rad}$ $\quad r = 10 \text{ft}$

$x = r\cos(\theta) = 10\cos(\pi/5) \approx 8.090169944$

$y = r\sin(\theta) = 10\sin(\pi/5) \approx 5.877852523$

$\boxed{(x, y) \approx (8.09, 5.88)}$

(b) Give Ron's x and y coordinates after 50 seconds.

$V = 2 \text{ ft/sec}$ $\quad r = 10 \text{ ft} \Rightarrow \omega = \frac{V}{r} = \frac{2}{10} \frac{\text{rad}}{\text{sec}} = \frac{1}{5} \text{ rad/sec}$

$\theta_0 = -\frac{\pi}{2} \text{ rad}$

$\theta = \omega t + \theta_0 = \frac{1}{5} \frac{\text{rad}}{\cancel{\text{sec}}} 50 \cancel{\text{sec}} - \frac{\pi}{2} \text{ rad} = 10 - \frac{\pi}{2} \text{ rad}$

$\theta = 10 - \pi/2 \approx 8.429203673 \text{ rad}$ $\quad r = 10 \text{ ft}$

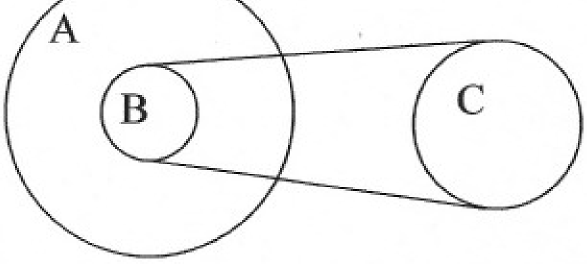
$x = r\cos(\theta) = 10\cos(10 - \pi/2) \approx -5.440211109$

$y = r\sin(\theta) = 10\sin(10 - \pi/2) \approx 8.390715291$

$\boxed{(x, y) \approx (-5.44, 8.39)}$

3. (10 points) Consider the following belt-and-wheel system with the three wheels A, B, and C. Wheel C has radius 7 inches and Wheel B has radius 4 inches. Wheel C has an angular speed of 10 revolutions/minute and Wheel A has a linear speed of 900 inches/minute. This situation is illustrated below.

Find the radius of Wheel A.



conversion: $10 \frac{\text{rev}}{\text{min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = 20\pi \frac{\text{rad}}{\text{min}}$

r		v	ω
r=	A	$900 \frac{\text{in}}{\text{min}}$	$35\pi \frac{\text{rad}}{\text{min}}$
r=4in	B	$140\pi \frac{\text{in}}{\text{min}}$	$35\pi \frac{\text{rad}}{\text{min}}$
r=7in	C	$140\pi \frac{\text{in}}{\text{min}}$	$20\pi \frac{\text{rad}}{\text{min}}$

② $\omega_B = \frac{V_B}{r_B} = \frac{140\pi}{4} = 35\pi \frac{\text{rad}}{\text{min}}$

① $V_C = \omega_C r_C$

$= 20\pi \cdot 7 = 140\pi \frac{\text{in}}{\text{min}}$

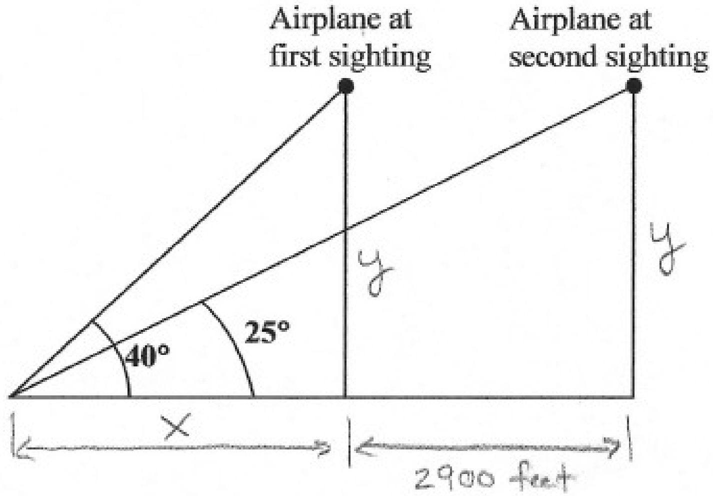
analysis: $V_B = V_C$, $\omega_A = \omega_B$

③ $r_A = \frac{V_A}{\omega_A} = \frac{900}{35\pi} \approx 8.185111359$ inches

$\boxed{8.19 \text{ inches}}$

4. (10 points) Grace is standing in her front yard and sees an airplane overhead. The plane is flying away from Grace at a constant height and a constant speed of 290 feet/second. When Grace sees the plane for the first time the angle measures 40 degrees. Grace sees the plane for the second time 10 seconds later and measures an angle of 25 degrees. (Hint: The plane travels 2900 feet between sightings.)

Find the height of the airplane.



$$\tan(25°) = \frac{y}{x+2900} \Rightarrow y = (x+2900)\tan(25°)$$

$$\tan(40°) = \frac{y}{x} \Rightarrow y = x\tan(40°)$$

combine $\Rightarrow$ $x\tan(40°) = (x+2900)\tan(25°)$

$$x(\tan(40°) - \tan(25°)) = 2900\tan(25°)$$

$$x = \frac{2900\tan(25°)}{\tan(40°)-\tan(25°)} \approx 3627.471369 \text{ feet}$$

$$y = x\tan(40°) = \frac{2900\tan(25°)\tan(40°)}{\tan(40°)-\tan(25°)}$$

$$\approx 3043.809888 \text{ feet}$$

$\boxed{\text{height} \approx 3043.81 \text{ feet}}$

5. (10 points)

a) Let $g(x)=\begin{cases} x+3\ , & \text{if } x<9 \\ 17\ , & \text{if } x\geq 9 \end{cases}$ and $h(x)=\begin{cases} -5\ , & \text{if } x<2 \\ x-1\ , & \text{if } x\geq 2. \end{cases}$

Find the multipart rule for $2g(x)-h(x)$.

g: $y=x+3$ [$x=9$ $y=17$

h: $y=-5$ [$x=2$ $y=x-1$

$$2g(x)-h(x)=\begin{cases} 2(x+3)-(-5), & \text{if } x<2 \\ 2(x+3)-(x-1), & \text{if } 2\leq x<9 \\ 2(17)-(x-1), & \text{if } x\geq 9 \end{cases}$$

$$=\begin{cases} 2x+11, & \text{if } x<2 \\ x+7, & \text{if } 2\leq x<9 \\ -x+35, & \text{if } x\geq 9 \end{cases}$$

b) Let $f(x)=\dfrac{x}{x+10}$.

Find $f^{-1}(x)$ and give ~~all vertical asymptotes~~ the domain of $f^{-1}(x)$.

$$y=\frac{x}{x+10} \qquad yx+10y=x$$

$$10y=x(1-y)$$

$$x=\frac{10y}{1-y}$$

$$\boxed{f^{-1}(x)=\frac{10x}{1-x}=\frac{-10x}{x-1}}$$

Domain

$$\boxed{x\neq 1}$$