1. (10 points) A circular puddle has radius 3 feet. Harry, the guinea pig, plans to walk through the puddle and cool off.

Harry is located 5 feet east and 4 feet north of the center of the puddle and he plans to walk directly toward the westernmost edge of the puddle. Harry walks at a constant speed of 1.3 ft/sec.

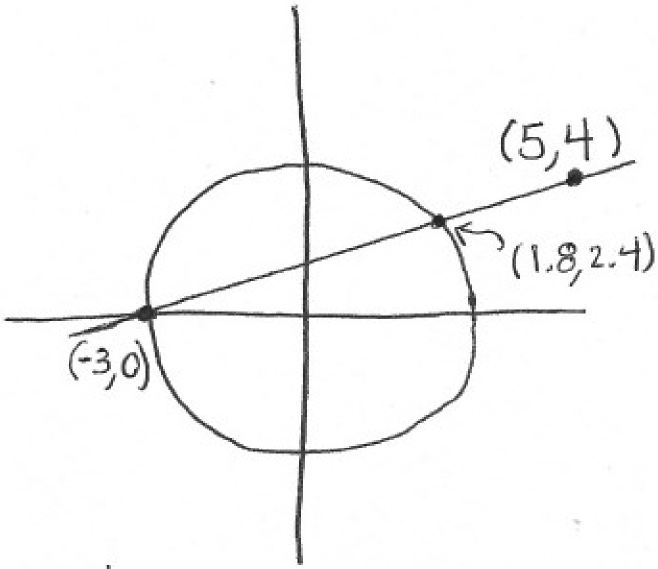
How long, in seconds, will it take Harry to first enter the puddle?

$\boxed{\text{circle}}$: $x^2+y^2=r^2$, $r=3$

$\boxed{x^2+y^2=9}$

$\boxed{\text{line}}$: $y=m(x-x_1)+y_1$, $m=\frac{4-0}{5--3}=\frac{1}{2}$

$\boxed{y=\frac{1}{2}(x+3)}$



$\boxed{\text{INTERSECTION}}$

$$x^2+\left[\frac{1}{2}(x+3)\right]^2=9$$

$$x^2+\frac{1}{4}(x^2+6x+9)=9$$

$$1.25x^2+1.5x+2.25=9$$

$$1.25x^2+1.5x-6.75=0$$

$\boxed{\text{Quadratic Formula}}$

$$x=\frac{-1.5\pm\sqrt{(1.5)^2-4(1.25)(6.75)}}{2(1.25)}$$

$$x=\frac{-1.5\pm\sqrt{36}}{2.5}$$

$$x=\frac{-1.5\pm 6}{2.5}$$

$x=-3$ or $\boxed{x=1.8}$

$y=0$ $\quad y=\frac{1}{2}(1.8+3)$

$\boxed{y=2.4}$

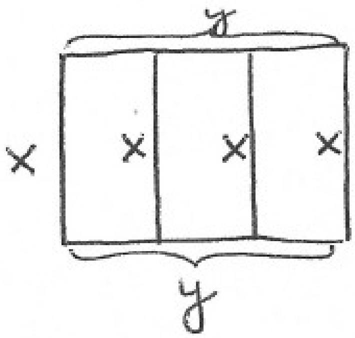
$\boxed{\text{TIME TO ENTER PUDDLE}}$

$$\text{time}=\frac{\text{dist}}{\text{speed}}=\frac{\sqrt{(5-1.8)^2+(4-2.4)^2}}{1.3 \text{ ft/sec}}\approx\boxed{2.752084 \text{ sec}}$$

$\boxed{2.75 \text{ seconds}}$

2. (10 points) Phil has 923 feet of fencing to make a rectangular enclosure. He also wants to use some fencing to split the enclosure into three parts with two interior fences that are parallel (this situation is illustrated below). What dimensions should the enclosure have to give the maximum possible total area?

$\boxed{\text{Label}}$ This could be labelled in other ways.



$\boxed{\text{Perimeter/Fencing Equation}}$

$$\boxed{4x+2y=923} \Rightarrow \quad 2y = 923-4x$$

$$\boxed{y = 461.5-2x}$$

$\boxed{\text{Area Function}}$

$$\text{Area} = xy$$

$$\boxed{A(x) = x(461.5-2x) = 461.5x-2x^2}$$

$\boxed{\text{Maximum}}$ This is a quadratic function with $a=-2$, $b=461.5$, and $c=0$.

$a=-2<0 \Rightarrow$ opens downward

So the vertex is the maximum.

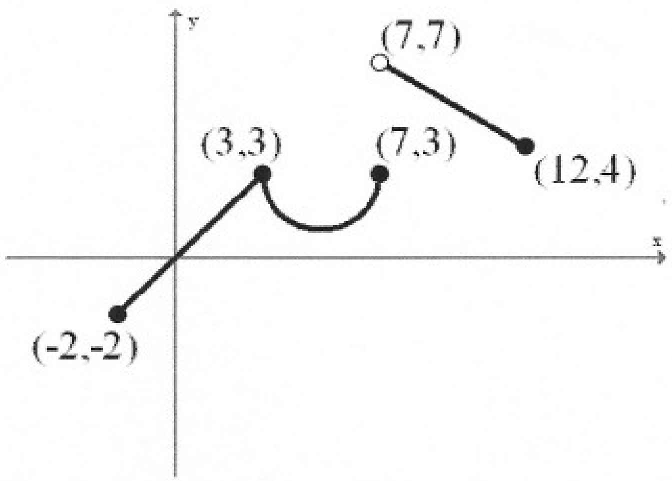
x-coord of vertex

$$h = -\frac{b}{2a} = -\frac{461.5}{2(-2)} = 115.375$$

$$y = 461.5-2(115.375) = 230.75$$

$$\boxed{x = 115.375 \text{ feet} \qquad y = 230.75 \text{ feet}}$$

3. (10 points) Consider the function, $f(x)$, given by the graph below which consists of two line segments and a lower semicircle.



a) Find the multipart formula for $y = f(x)$.

$\boxed{\text{line 1}}$: $m = \frac{3 - -2}{3 - -2} = 1$ $\quad y = x$ $\quad \boxed{\text{line 2}}$ $m = \frac{4-7}{12-7} = -\frac{3}{5}$ $\boxed{y = -\frac{3}{5}(x-7)+7}$

$\boxed{\text{Semicircle}}$ $y = k - \sqrt{r^2 - (x-h)^2}$ $\quad (h,k) = (5,3),\ r = 2$

$y = 3 - \sqrt{4-(x-5)^2}$

$$\boxed{f(x) = \begin{cases} x, & \text{if } -2 \le x \le 3 \\ 3 - \sqrt{4-(x-5)^2}, & \text{if } 3 \le x \le 7 \\ -\frac{3}{5}(x-7)+7, & \text{if } 7 < x \le 12 \end{cases}}$$

b) Compute the value $f(4)$.

$$f(4) = 3 - \sqrt{4-(4-5)^2}$$

$$= 3 - \sqrt{3} \approx 1.267949192$$

$$\boxed{1.27}$$

4. (10 points) Let $f(x) = 3 - x^2$ and $g(x) = \begin{cases} 6x^2 & , \text{if } x < 2 \\ x + 10 & , \text{if } x \geq 2 \end{cases}$.

a) Evaluate and simplify $\frac{f(x+a) - f(x)}{a}$. (Simplify as much as possible)

$$= \frac{[3 - (x+a)^2] - [3 - x^2]}{a} = \frac{3 - (x^2 + 2ax + a^2) - 3 + x^2}{a}$$

$$= \frac{\cancel{3} - \cancel{x^2} - 2ax - a^2 - \cancel{3} + \cancel{x^2}}{a} = \frac{-2ax - a^2}{a} = \boxed{-2x - a}$$

b) Give a multipart formula for the composition $f(g(x))$.

$\xrightarrow{x<2} 6x^2 \xrightarrow{f(6x^2)} 3 - (6x^2)^2 = 3 - 36x^4$

X $\xrightarrow{x \geq 2} x+10 \xrightarrow{f(x+10)} 3 - (x+10)^2 = 3 = ($

$$\boxed{f(g(x)) = \begin{cases} 3 - 36x^4, & \text{if } x < 2 \\ 3 - (x+10)^2, & \text{if } x \geq 2 \end{cases}}$$

c) Give all values of x that satisfy $g(x) = 6$.

$6 = 6x^2$, $x < 2$ or $6 = x + 10$, if $x \geq 2$

$1 = x^2$

$x = \pm 1$ both in domain

$\cancel{x = -4}$ not in domain

$\boxed{x = -1, \quad x = +1}$