MATH 120 – EXAM I Hints and Answers
Version 1
Autumn 2005

1. ANSWER: $4x + 2h$

2. HINT: The quadratic equation $ax^2 + bx + c = 0$ will have exactly one solution if $b^2 - 4ac = 0$.
   ANSWER: $\alpha = \pm\sqrt{3}$

3. HINT: With a coordinate system imposed so that the origin is at Pam's apartment building, the circle that defines the range of the "hotspot" is $(x - 3)^2 + (y - 0.25)^2 = 0.3125^2$. (The radius of the circle is 1650 feet, which is 0.3125 miles.) The circle intersects the x-axis at $x = 3 \pm 0.1875$. So, Pam is in the circle for a distance of $2 \times 0.1875 = 0.375$ miles. Use the fact that the bus travels 40 mph to find the amount of time she is in the circle in hours and convert to minutes.

   ANSWER: 0.5625 minutes

4. (5 points each)

   (a) HINT: Enrollment reaches a maximum of 3600 students at time $t = 10$. This means that the vertex of the parabola is at the point $(10, 3600)$. That gives the values of h and k. So, $C(t) = a(t - 10)^2 + 3600$. To find a, use the fact that $C(0) = a(0 - 10)^2 + 3600$ and $C(0) = 852$. That gives the equation $100a + 3600 = 852$. Solve for a.
   ANSWER: $h = 10$, $k = 3600$, $a = -27.48$

   (b) HINT: Find the equation of the line through the points $(0, 200)$ and $(10, 950)$.
   ANSWER: $L(t) = 75t + 200$

   (c) HINT: The right-handed population is the difference between the total enrollment and the left-handed population. That is, $R(t) = C(t) - L(t)$, where $C(t) = -27.48(t - 10)^2 + 3600$ and $L(t) = 75t + 200$. After some simplification, $R(t) = -27.48t^2 + 474.6t + 652$. This is a quadratic, whose graph is a parabola that opens down. It is largest at its vertex.
   ANSWER: $t = 8.64$ years

5. (5 points each)

   (a) HINT: The y-intercept of the graph is $f(0) = (0)^2 + 6(0) + 8 = 0$. So, the line segment goes through the points $(0, 8)$ and $(9, 0)$. Find its equation.
   ANSWER: $-\frac{8}{9}x + 8$

   (b) HINT: For the range, we need to know the lowest and highest y-values on the graph of f. The high value occurs at either $x = -6$ or $x = 0$. It turns out that $f(-6)$ and $f(0)$ are both 8. The low value occurs at either $x = 10$ or the vertex of the parabola. The vertex of the parabola occurs at $x = -3$. (Why?) Further, $f(-3) = (-3)^2 + 6(-3) + 8 = -1$ and $f(10) = -\frac{8}{9}(10) + 8 = -\frac{8}{9}$. So, the low point on the graph has y-value -1.
   ANSWER: $D = \{x | -6 \le x \le 10\}$; $R = \{y | -1 \le y \le 8\}$

   (c) HINT: $(g \circ f)(x) = \sqrt{f(x) - 3}$. The domain of this function consists of all values of x that are in the domain of f and such that $f(x) - 3 \ge 0$. That is, the domain of $g \circ f$ consists of all values of x such that $-6 \le x \le 10$ and $f(x) \ge 3$. From the graph of f, I can see that $f(x) = 3$ in three places. Setting $x^2 + 6x + 8 = 3$ gives me two of these places: $x = -5$ and $x = -1$; setting $-\frac{8}{9}x + 8 = 3$ gives me the other. From the graph, I can see that $f(x) \ge 3$ if $-6 \le x \le -5$ or $-1 \le x \le \frac{45}{8}$.
   ANSWER: $D_{g \circ f} = [-6, -5] \cup [-1, \frac{45}{8}]$