1. (10 points) The fans of the local Mudville baseball team only seem to care about homeruns. When the team hits 50 homeruns in a season, the attendance is 6 million fans. When the team hits 100 homeruns in a season, the attendance is 13 million fans. The more homeruns the team hits, the closer and closer the attendance gets to the capacity of 20 million fans.

(a) Find the linear-to-linear model that gives attendance as a function of homeruns.

$$y = \frac{ax+b}{x+d}$$

① $x = 50 \Rightarrow y = 6$

② $x = 100 \Rightarrow y = 13$

③ $y = 20$ horiz. asymptote

③ $\Rightarrow \boxed{a = 20}$

① $\Rightarrow 6 = \frac{20(50)+b}{50+d} \Rightarrow \boxed{6(50+d) = 1000+b} \Rightarrow 300+6d = 1000+b$

$b = \underbrace{6d-700}$

② $\Rightarrow 13 = \frac{20(100)+b}{100+d} \Rightarrow \boxed{13(100+d) = 2000+b}$

$$1300+13d = 2000+6d-700$$

$$7d = 0 \qquad \boxed{d=0}$$

$$b = 6d-700 = 6(0)-700 \qquad \boxed{b=-700}$$

$$\boxed{y = \frac{20x-700}{x}}$$

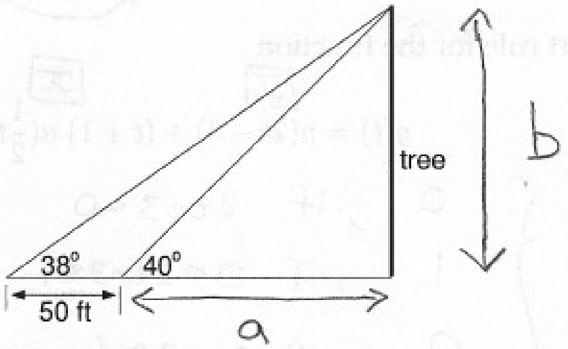
(b) Using your model, how many homeruns does the team have to hit to get exactly 18 million fans to attend?

$$18 = \frac{20x-700}{x}$$

$$18x = 20x-700$$

$$700 = 2x \qquad \boxed{x = 350 \text{ homeruns}}$$

2. (10 points) You are standing in a clearing at a certain distance from the base of a tall tree. You observe that the top of the tree makes an angle of 40° above the level ground at your current location. You move back 50 feet and observe that the top of the tree makes an angle of 38° above the level ground at the new location. This situation is illustrated below.



Find the height of the tree to the nearest foot.

① $\tan(40°) = \dfrac{b}{a} \Rightarrow b = a\tan(40°)$

② $\tan(38°) = \dfrac{b}{50+a} \Rightarrow b = (50+a)\tan(38°)$

$\tan(40°) \approx 0.8390996311 77$

$\tan(38°) \approx 0.7812856265 07$

[I] Using exact values

$a\tan(40°) = (50+a)\tan(38°)$

$a\tan(40°) = 50\tan(38°) + a\tan(38°)$

$a\left[\tan(40°) - \tan(38°)\right] = 50\tan(38°)$

$a = \dfrac{50\tan(38°)}{\tan(40°) - \tan(38°)}$

[II] Using numerical values

$a(0.8390996) = (50+a)(0.7812856)$

$a(0.8390996) = 39.06428 + 0.7812a$

$0.057814004671a = 39.06428133$

$a = \dfrac{39.0642813253}{0.057814004671}$

$a = 675.68890182$

$b = a\tan(40°) \approx 566.970308 \approx \boxed{567 \text{ feet}}$

3. (10 points) Let $u(t)$ be the unit step function,

$$u(t) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } 0 \le x \le 1 \\ 0 & \text{if } x > 1 \end{cases}$$

$\boxed{\text{typo}}$

These should be t's

$$\boxed{u(t) = \begin{cases} 0, & \text{if } t<0 \\ 1, & \text{if } 0 \le t \le 1 \\ 0, & \text{if } t>1 \end{cases}}$$

Write the multipart rule for the function

$$g(t) = \underbrace{u(2t-3)}_{\boxed{\text{I}}} + \underbrace{(t+1)\, u(\tfrac{1}{2}t - 1)}_{\boxed{\text{II}}}$$

$\boxed{\text{I}}$

$$u(2t-3) = \begin{cases} 0, & \text{if } 2t-3<0 \\ 1, & \text{if } 0 \le 2t-3 \le 1 \\ 0, & \text{if } 2t-3>1 \end{cases} \leftarrow \quad 3 \le 2t \le 4, \quad \tfrac{3}{2} \le t \le 2$$

$$u(2t-3) = \begin{cases} 0, & \text{if } t<\tfrac{3}{2} \\ 1, & \text{if } \tfrac{3}{2} \le t \le 2 \\ 0, & \text{if } t>2 \end{cases}$$

$y=0$ [$y=1$] $y=0$

$t=\frac{3}{2}$ $\quad t=2$

$\boxed{\text{II}}$

$$(t+1)u(\tfrac{1}{2}t-1) = \begin{cases} (t+1)\cdot 0, & \text{if } \tfrac{1}{2}t-1<0 \\ (t+1)\cdot 1, & \text{if } 0 \le \tfrac{1}{2}t-1 \le 1 \\ (t+1)\cdot 0, & \text{if } \tfrac{1}{2}t-1>1 \end{cases} \leftarrow \quad 1 \le \tfrac{1}{2}t \le 2, \quad 2 \le t \le 4$$

$$(t+1)u(\tfrac{1}{2}t-1) = \begin{cases} 0, & \text{if } t<2 \\ t+1, & \text{if } 2 \le t \le 4 \\ 0, & \text{if } t>4 \end{cases}$$

$y=0$ [$y=t+1$] $y=0$

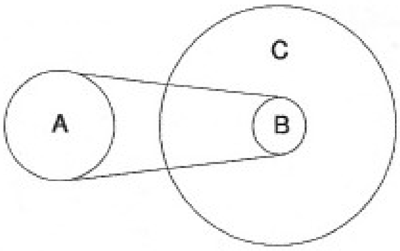
$t=2$ $\quad t=4$

$\boxed{\text{III}}$ $\boxed{\text{Add}}$

$$u(2t-3) + (t+1)u(\tfrac{1}{2}t-1) = \begin{cases} 0+0, & \text{if } t<\tfrac{3}{2} \\ 1+0, & \text{if } \tfrac{3}{2} \le t < 2 \\ 1+(t+1), & \text{if } t=2 \\ 0+(t+1), & \text{if } 2<t\le 4 \\ 0+0, & \text{if } t>4 \end{cases}$$

$$= \begin{cases} 0, & \text{if } t<\tfrac{3}{2} \\ 1, & \text{if } \tfrac{3}{2} \le t < 2 \\ t+2, & \text{if } t=2 \\ t+1, & \text{if } 2<t\le 4 \\ 0, & \text{if } t>4 \end{cases}$$

4. (10 points) You are riding a bicycle along a level road. Assume each wheel is 15 inches in radius and the front sprocket has radius 8 inches. The front sprocket, A, the rear sprocket B, and the rear wheel, C, are shown below.



If the front sprocket has angular speed of 200 revolutions per minute and the speed of the bike is 40 miles per hour, what is the radius of the rear sprocket?

		V	ω
$r = 8$ in	A	(2) $3200\pi \frac{\text{in}}{\text{min}}$	(1) $400\pi \frac{\text{rad}}{\text{min}}$
$r =$	B	(3) $3200\pi \frac{\text{in}}{\text{min}}$	(3) $2816 \frac{\text{rad}}{\text{min}}$
$r = 15$ in	C	(1) $42240 \frac{\text{in}}{\text{min}}$	(2) $2816 \frac{\text{rad}}{\text{min}}$

$V = \omega r = (400\pi)(8)$

$\omega = \frac{V}{r} = \frac{42240}{15}$

conversion: $200 \frac{\text{rev}}{\text{min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = 400\pi \frac{\text{rad}}{\text{min}} = \omega_A$

$40 \frac{\text{mi}}{\text{hr}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \cdot \frac{12 \text{ in}}{1 \text{ ft}} = 42240 \frac{\text{in}}{\text{min}} = V_c$

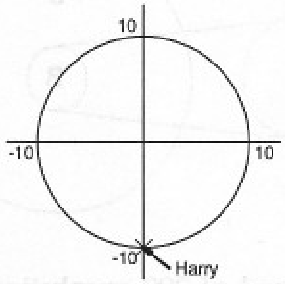
A & B have same linear speed

B & C have same angular speed

$$r = \frac{V}{\omega} = \frac{3200\pi}{2816} \approx 3.569991652 \text{ inches}$$

$$\boxed{3.57 \text{ inches}}$$

5. (10 points) Harry is standing on the far southern outer edge of a merry-go-round of radius 10 feet. The merry-go-round is rotating counterclockwise with an angular speed of 15 revolutions per minute. Below we give a figure of this situation and we impose a coordinate system with the origin at the center of the merry-go-round.



Give the (x, y) coordinates of Harry after 2 seconds.

$x = r\cos(\theta)$ $\qquad$ θ = angle in standard position

$y = r\sin(\theta)$

r = radius

$r = 10$ feet $\qquad \omega = 15 \frac{\text{rev}}{\text{min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = 30\pi \frac{\text{rad}}{\text{min}}$

$t = 2 \text{ sec} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = 0.0\overline{3} \text{ min} = \frac{1}{30} \text{ min}$

$\theta = \omega t$ = angle swept out from Harry's start location

$30\pi \frac{\text{rad}}{\text{min}} \cdot 0.0\overline{3} \text{ min} = \boxed{\pi \text{ radians}}$

Harry starts at $-\pi/2$ radians, so his angle in standard position is $\omega t + \theta_0 = \pi + -\pi/2 = \boxed{\pi/2 \text{ radians}}$

$$\boxed{\begin{aligned} x &= 10\cos(\pi/2) = 0 \\ y &= 10\sin(\pi/2) = 10 \end{aligned}}$$