Math 120 D and E - Autumn 2005
Exam 1
October 20, 2005

Name: Instructor Key

Section: Version 1

Student ID Number: _____

TA's Name: _____

1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

- You are allowed to use a calculator and one **hand-written** 8.5 by 11 inch page of notes.
- Check that your exam contains all the problems listed above.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit.
- Write your answers in the specified locations. Unless otherwise indicated, you may round your **final answer** to two digits after the decimal.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so. If you still need more paper, please ask for some.
- Raise your hand if you have a question.
- Put your name on your sheet of notes and turn it in with the exam.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 50 minutes to complete the exam.

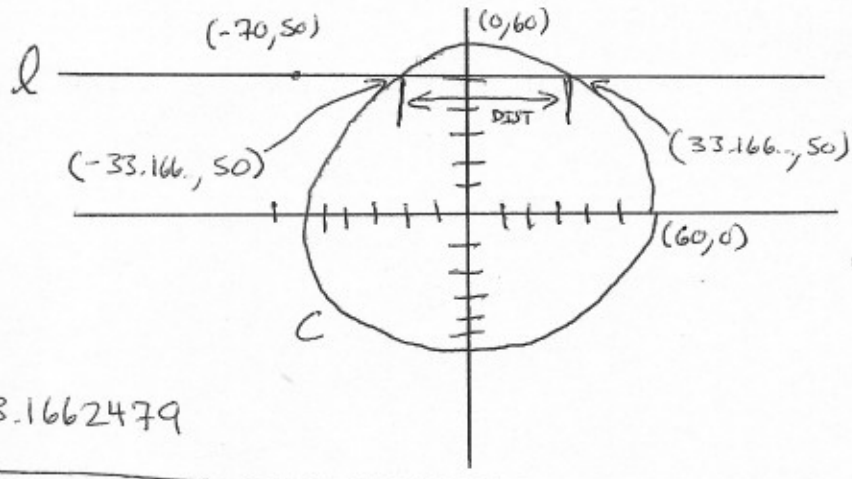
GOOD LUCK!

Version 1

1. (10 points) An airplane is flying east at 150 mph. An airport is located 70 miles east and 50 south of the plane's current location. The airport has radar for detecting any plane within 60 miles. How long, in minutes, will the airplane be within the radar region?

ℓ: $y = 50$

C: $x^2 + y^2 = 60^2$



INTERSECTION POINTS

$x^2 + 50^2 = 60^2$

$x^2 = 3600 - 2500 = 1100$

$x = \pm\sqrt{1100} \approx \pm 33.1662479$

DISTANCE

dist = $33.1662479 - -33.1662479 = 66.33249581$ mi $\quad (2\sqrt{1100})$

TIME

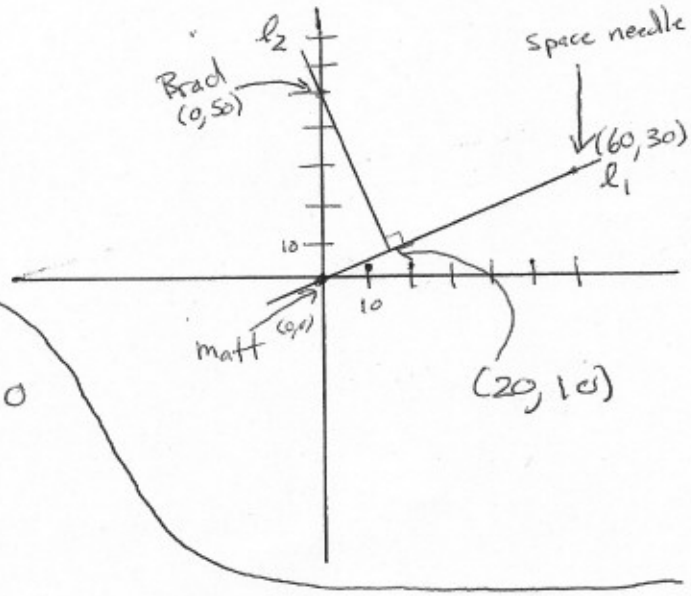
$\text{time} = \frac{\text{dist}}{\text{speed}} = \frac{66.33249581 \text{ mi}}{150 \text{ mph}} = 0.442216638 \text{ hrs} \quad \frac{60 \text{ min}}{1 \text{ hr}}$

$= 26.53299832$ min

$= \boxed{26.53 \text{ min}}$

Version 1

2. (10 points) Matt and Brad are meeting by the space needle. Matt is located 60 feet west and 30 feet south of the space needle. Brad is standing 50 ft north of Matt. If Matt walks directly toward the space needle, at what location will Matt be closest Brad?



l_1 : slope $= \frac{30-0}{60-0} = \frac{1}{2}$

$y = \frac{1}{2}(x-0)+0 = \frac{1}{2}x$

l_2 : slope $= -\frac{1}{(1/2)} = -2$

$y = -2(x-0)+50 = -2x+50$

INTERSECTION POINT

$\frac{1}{2}x = -2x+50$

$2.5x = 50$

$x = \frac{50}{2.5} = 20$ $\qquad y = \frac{1}{2}x = \frac{1}{2}(20) = 10$

$x = 20$

$y = 10$

Note that the coordinates would be different if we moved the origin.

For example, if the space needle was the origin then the coordinates are

$x = 20 - 60 = -40$

$y = 10 - 30 = -20$

Version 1

3. (10 points) Let $f(x) = |x| + 7$ and $g(x) = 3x + 6$.

(a) Write the multipart rule for $f(g(x))$.

$$f(g(x)) = |3x+6|+7 = \begin{cases} (3x+6)+7, & 3x+6 \geq 0 \\ -(3x+6)+7, & 3x+6 < 0 \end{cases}$$

$$= \begin{cases} 3x+13, & x \geq -2 \\ -3x+1, & x < -2 \end{cases}$$

$3x \geq -6$

$x \geq -2$

note they can also write

$$= \begin{cases} 3x+13, & x \geq -2 \\ -3x+1, & x \leq -2 \end{cases}$$

$$= \begin{cases} 3x+13, & x > -2 \\ -3x+1, & x \leq -2 \end{cases}$$

since $f(g(x))$ is cont. at $x=-2$.

(b) Find all solutions to the equation

$$f(g(x)) = 13.$$

$x \geq -2$

$3x+13 = 13$

$3x = 0$

$\boxed{x = 0}$ in domain ✓

OR

$x < -2$

$-3x+1 = 13$

$-3x = 12$

$\boxed{x = -4}$ in domain ✓

Version 1

4. (10 points) Suppose you are in charge of ticket sales at a football stadium. You found if you charge \$10 dollars a ticket, you sell 1000 tickets and if you charge \$13.50 dollars a ticket, you sell 970 tickets. If the tickets sold is a linear function of price, what price should you charge to get the most money?

$f(x)$ = total money taken in when price = x

$= (\text{price})(\text{quantity sold})$

$= x(\text{quantity sold})$

$= x[m(x-x_1)+y_1]$

$(10, 1000) \quad (13.5, 970)$

$\text{SLOPE} = m = \frac{970-1000}{13.5-10} \approx -8.571428571$

$f(x) = x[-8.571428571(x-10)+1000]$

$= -8.571x^2 + 1085.714x$

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find the max by either ① $x = -\frac{b}{2a}$ or ② Completing the square

① $x = -\frac{b}{2a} = -\frac{1085.714}{2(-8.571)}$

$= 63.\overline{3}$

price = \$63.33

(it is closer to say \$63.34)

② $-8.571x^2 + 1085.714x = a(x-h)^2+k$

$= ax^2 - 2ahx + ah^2 + k$

$a = -8.571$

$-2ah = 1085.714 \quad h = \frac{1085.714}{2(8.571)}$

$= 63.\overline{3}$

Version 1

5. (10 points) Let $f(x) = 3x^2 - 4x$. Assuming $h \neq 0$, simplify the expression

$$\frac{f(x+h) - f(x)}{h}$$

as much as possible.

$$\frac{[3(x+h)^2 - 4(x+h)] - [3x^2 - 4x]}{h}$$

$$= \frac{3(x^2 + 2hx + h^2) - 4x - 4h - 3x^2 + 4x}{h}$$

$$= \frac{3x^2 + 6hx + 3h^2 - 4x - 4h - 3x^2 + 4x}{h}$$

$$= \frac{6hx + 3h^2 - 4h}{h}$$

$$= \frac{(6x + 3h - 4)h}{h} = \boxed{6x + 3h - 4}$$