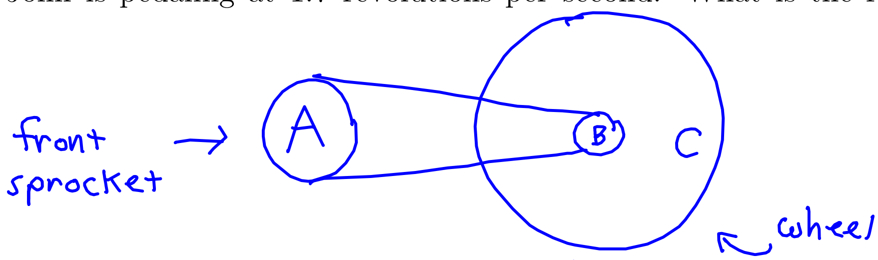
Math 120 Exam II - Version One.

Fall 2004, Thursday November 18th. Dr. Schneider.

There were two versions of the exam. To see which version you have, look at problem one. If the front sprocket has a radius of 6 inches, you have version one (this version). If the radius is 5 inches, you have version two. Math 120B had a different problem 3 then Math 120A or Math 120D. Solutions for both problems are in this solution guide.

Problem 1 [12 pts] Suppose John is riding a bike at 25 miles and hour. The wheels of the bike are 28 inches in diameter. The front sprocket of the bike has a radius of 6 inches. John is pedaling at 1.7 revolutions per second. What is the radius of the rear sprocket?



Front Sprocket:

$r_A = 6''$

$\omega_A = 1.7 \frac{rev}{min} = 10.681 \frac{rad}{sec}$

$V_A = r_A \omega_A$

$V_A = 64.0885 \frac{in}{sec}$

Back Wheel:

$r_c = 14''$

$V_c = 25 \frac{miles}{hour} = 440 \frac{in}{sec}$

$\omega_c = \frac{V_c}{r_c}$

$\omega_c = 31.4286 \frac{rad}{sec}$

Rear Sprocket:

$\omega_B = \omega_c$ $\qquad r_B = ?$

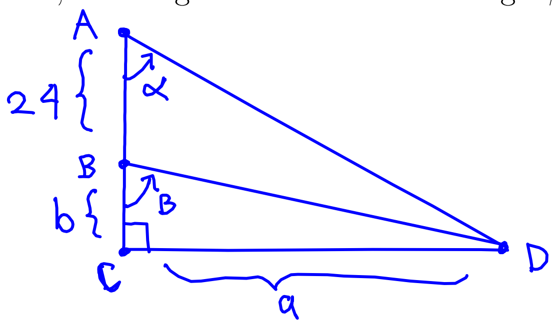
$V_B = r_B \omega_c = r_B \, 31.4286 \, \cancel{rad} \frac{in}{sec}$

set $V_A = V_B$

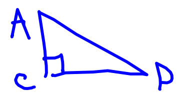
$r_B \, 31.4286 = 64.00885$

$r_B = 2.039$ inches

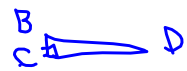
Problem 2 [12 pts] In the figure below, suppose that the length of line segment AB is 24 feet, that angle α is 32° and that angle β is 54°. Find the length of line segment CD.



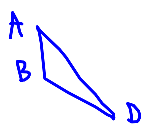
There are two right triangles:



and



Note,



is NOT a right triangle, so it is incorrect to do things like $\sin\alpha = \frac{BD}{AD}$.

From the two triangles, one gets

$$\tan(\alpha) = \frac{a}{b+24} \qquad \tan(\beta) = \frac{a}{b}$$

$$(b+24)\tan(\alpha) = a \qquad b\tan(\beta) = a$$

set these equal, or eliminate a:

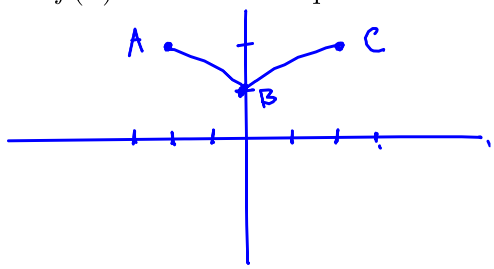
$$b\tan(\alpha) + 24\tan(\alpha) = b\tan(\beta)$$

$$24\tan(\alpha) = b[\tan(\beta) - \tan(\alpha)]$$

$$\frac{24\tan(\alpha)}{\tan(\beta) - \tan(\alpha)} = b$$

So $b = 19.9556$ and thus $a = b\tan(54°) = 27.4665$ feet

Problem 3 [12 pts] The graph of the function $f(x)$ is given in the figure below. The coordinates of the labeled points are $A = (-2, 2)$, $B = (0, 1)$, and $C = (2, 2)$. The minimum for $f(x)$ occurs at the point B.



Let $g(x) = 3f(\frac{1}{2}(x + 3)) - 1$.

(a) [6 pts] What is the domain of $g(x)$?

Domain of f: $[-2, 2]$

If x is in Domain of g(x), then $-2 \leq \frac{1}{2}(x+3) \leq 2$

solving for x one gets: $-4 \leq x+3 \leq 4$

$$\boxed{-7 \leq x \leq 1}$$

(b) [6 pts] What are the coordinates for the minimum of $g(x)$?

old minimum of f: $(0,1)$

x coord for minimum of g: $\frac{1}{2}(x+3) = 0$

$x + 3 = 0$

$$\boxed{x = -3}$$

~~y coord for min of g:~~ $\cancel{\frac{1}{3}\cdot 1 - 1 =}$

y coord for min of g: $3 \cdot 1 - 1 = 2$

answer: $\boxed{(-3,2)}$

Problem 3 [12 pts] Let u(t) be the basic step function:

$$u(t) = \begin{cases} 0 \text{ if } t < 0 \\ 1 \text{ if } 0 \le t \le 1 \\ 0 \text{ if } 1 < t \end{cases}$$

a) [4 pts] Find the multipart rule for $f(t) = \frac{2}{5}u(\frac{1}{2}(t-2))$

if $0 \le \frac{1}{2}(t-2) \le 1$ then $2 \le t \le 4$

answer $f(t) = \begin{cases} 0 & \text{if } t < 2 \\ 2/5 & 2 \le t \le 4 \\ 0 & 4 < t \end{cases}$

b) [5 pts] Find the multipart rule for $g(t) = (t+2)u(\frac{1}{3}(t+1))$.

if $0 \le \frac{1}{3}(t+1) \le 1$ + $-1 \le t \le 2$

then multiply y values, $\begin{smallmatrix}0\\1\\0\end{smallmatrix}$ by $(t+2)$:

$g(t) = \begin{cases} 0 & t < -1 \\ t+2 & -1 \le t \le 2 \\ 0 & 2 < t \end{cases}$

c) [3 pts] Let $h(t) = g(t) - f(t)$. Find the following

$$h(-1) = \qquad h(2) = \qquad h(4) =$$

$h(-1) = g(-1) - f(-1)$. To figure out $g(-1)$, look at the rules above.

When $-1 \le t \le 2$ use the rule $g(t) = t + 2$

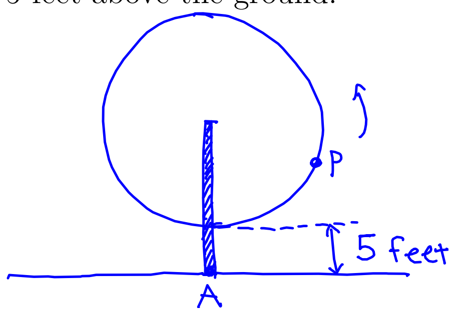
Therefore $g(-1) = 1$ so $h(-1) = 1$

similarly $f(-1) = 0$

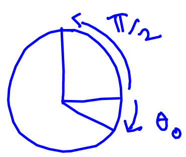
likewise $h(2) = 4 - \frac{2}{5} = 3\frac{3}{5}$

$h(4) = 0 - \frac{2}{5} = -\frac{2}{3}$

Problem 4 [12 pts] Suppose you are on a ferris wheel that makes two complete revolutions, in a counter clockwise direction, every three minutes. At time $t = 0$, you are at some point P on the ferris wheel. It takes 25 seconds for you to reach the very top of the ferris wheel. The ferris wheel has a diameter of 80 feet. Assume the bottom of the ferris wheel is 5 feet above the ground.



(a) [8 pts] What is your height above the ground, as a function of t, where t is in minutes?



$\omega = \frac{2 \text{ rev}}{3 \text{ min}} = \frac{4\pi}{3} \frac{\text{rad}}{\text{min}}$ or $4.1888 \frac{\text{rad}}{\text{min}}$

$\theta = \theta_0 + \omega t$

when $t = \frac{25}{60}$ minutes, $\theta = \frac{\pi}{2}$ so

$\frac{\pi}{2} = \theta_0 + \frac{4\pi}{3}\left(\frac{25}{60}\right) \Rightarrow \theta_0 = -\frac{\pi}{18}$ or $-.1745$ rad

height $= 40 \sin\left(-\frac{\pi}{18} + \frac{4}{3}\pi t\right) + 45$

radius of wheel (the 40); height of wheel center above ground (the 45)

(b) [4 pts] After 55 seconds, what is your distance from point A? (Note that point A is on the ground, at the base of the ferris wheel.)

origin is at A

$x = 40 \cos\left(-\frac{\pi}{18} + \frac{4}{3}\pi t\right)$

$y = 40 \sin\left(-\frac{\pi}{18} + \frac{4}{3}\pi t\right) + 45$

when $t = \frac{55}{60}$ angle $-\frac{\pi}{18} + \frac{4}{3}\pi t$ is $\frac{7\pi}{6}$ rad

so point on wheel is $x = 40\cos\left(\frac{7\pi}{6}\right)$ $y = 40 \sin\left(\frac{7\pi}{6}\right) + 45$

or $(-34.641, 25)$

so distance is $\sqrt{34.641^2 + 25^2} = 42.72$ feet