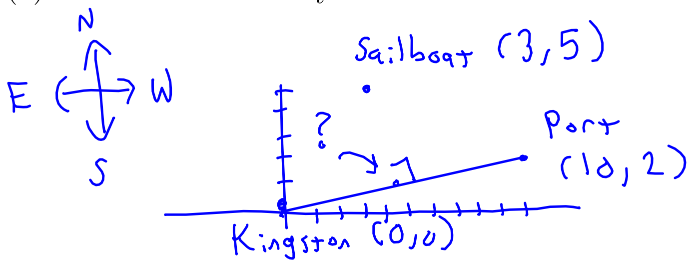
Math 120A,B, E Exam I Version 1 Solutions

Check which version of the exam you have by looking at problem one. If it starts out with 5 miles north, it is this version (version 1). If it starts with 7 miles north, it is version two (and you are looking at the wrong solutions!)

Problem 1. Johnny's sailboat is sitting still in the water 5 miles North and 3 miles east of the city of Kingston. A Ferry has just left Kingston and is traveling on a straight line to a port city that is 10 miles east and 2 miles north of Kingston. The Ferry travels at 13 mph.
(a) Where will the ferry be when it is closest to Johnny's sailboat?



Kingston to Port City line: $m = \frac{2}{10} = \frac{1}{5}$ $\quad y = \frac{1}{5}x$

Perpendicular line: $m = -5$ pt on line $(3,5)$

$$y = -5(x-3)+5 = -5x+20$$

Intersection: $\frac{1}{5}x = -5x + 20$

$$5.2x = 20 \qquad x = 3.846$$

$$y = \frac{1}{5}x = .769$$

Closest point is 3.846 miles east & .769 miles North of Kingston.

(b) After the ferry leaves Kingston, how long will it take before the ferry reaches this location that is closest to Johnny's sailboat?

distance of closest point from Kingston: $\sqrt{3.846^2 + .769^2}$

which is 3.92 miles

$$t = \frac{d}{r} = \frac{3.92 \text{ miles}}{13 \text{ mph}} = .302 \text{ hours}$$

(c) If Johnny has a radar that lets him detect any ships within 4 miles of his position, will he be able to detect the ferry?

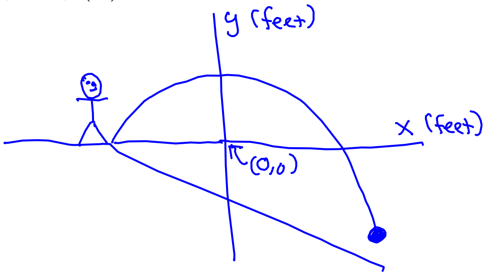
(3, 5)

distance of closest point to Sailboat: pts (3.846, .769)

$\sqrt{.846^2 + 4.231^2} = 4.315$ miles

This is not within 4 miles of Sailboat. $\boxed{\text{NO}}$

Problem 2. Berry is standing 10 feet west of the origin in the coordinate system shown below. Starting at this point, the ground slopes downward, with the ground level being given by $y = \frac{-4}{9}(x + 10)$. Berry kicks a ball westward and the ball follows the graph of $y = f(x) = -x^2 + 2x + 120$.



(a) Where does the ball land on the slope?

You need to find where $y = -x^2 + 2x + 120$ and $y = -\frac{4}{9}(x + 10)$ intersect.

$$-x^2 + 2x + 120 = -\frac{4}{9}(x + 10)$$
$$x^2 - 2.444x - 124.444 = 0$$

quadratic formula gives $x = -10$ and $x = 12.444$. The y coordinate is $y = -\frac{4}{9}(12.444+10) = -9.973$.
The answer is $(12.444, -9.973)$.

(b) What is the maximum height of the ball above the slope?

height: $-x^2 + 2x + 120 - \left(-\frac{4}{9}(x+10)\right) =$

$h(x) = -x^2 + 2.444x + 124.444$

vertex: $x = \frac{-b}{2a} = 1.222$

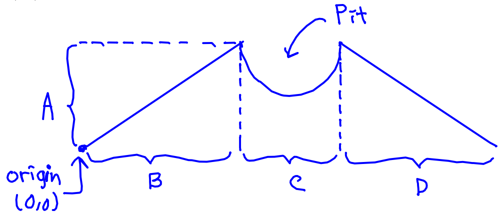
$y = h(1.222) = -1.222^2 + 127.432 = 125.938$

max height above slope is 125.938 feet.

Note, it is not correct to find the vertex of the parabola $y = f(x) = -x^2 + 2x + 120$, which is $(1, 120.75)$. Nor is it correct to take this height of 120.75 and subtract from it $-\frac{4}{9}(1 + 10) = -4.8889$, even though the answer is very close.

Problem 3. The vertical cross section for a volcano is given in the following picture. The origin is at the lower left corner of the picture. The sides of the volcano are line segments, and the pit of the volcano is a semicircle (half of a circle). The lengths in the diagram are $A = 3.96$ kilometers, $C = 2.13$ kilometers, and both B and D are 4.57 kilometers.

(a) Find a multipart function that models the vertical cross section of this volcano.



line one slope: $\frac{A}{B} = \frac{3.96}{4.57} = .869$ 

first part of function $y = .869x$

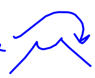
circle – radius is $\frac{1}{2}C = \frac{1}{2}(2.13) = 1.065$

center has $x = B + \frac{1}{2}C = 4.57 + 1.065 = 5.635$

$y = A = 3.96$

circle is $(x - 5.635)^2 + (y - 3.96)^2 = 1.065^2$

solve for lower semicircle: $y = 3.96 - \sqrt{1.065^2 - (x - 5.635)^2}$

last line  slope $-\frac{A}{B} = -.869$ point on line 

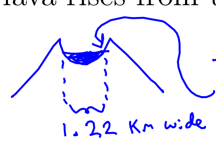
$x = B + C + D = 11.27$

$y = 0$

all together

$$f(x) = \begin{cases} .869x & \text{if } 0 \le x \le 4.57 \\ 3.96 - \sqrt{1.134 - (x - 5.635)^2} & \text{if } 4.57 \le x \le 6.7 \\ -.869(x - 11.27) & \text{if } 6.7 \le x \le 11.27 \end{cases}$$

(b) Suppose the pit of the volcano is filling with lava. The lava rises at a rate of 30 meters per minute. How long will it take for the lava in the pit to be 1.22 kilometers wide? (The lava rises from the bottom of the pit, not the bottom of the volcano).



the x coordinate of top of lava against pit is $x =$ 

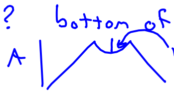
go to circle center $+ \frac{1}{2}(1.22)$

$x = 4.57 + 1.065 + .61 = 6.245$

Then y coord, from second part (circle part) of multipart function is

$$3.96 - \sqrt{1.134 - (6.245 - 5.635)^2} = 3.087$$

lava height ? bottom of pit is at A – radius



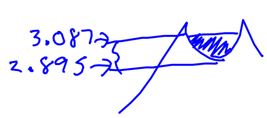
$3.96 - 1.065$

$= 2.895$

lava goes from 2.895 to 3.087

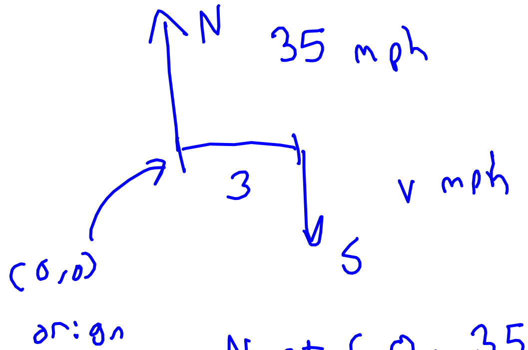
or .192 Km = 192 meters.

time $= \frac{192}{30} = 6.4$ minutes.



Problem 4. Two cars start out 3 miles apart, one due east of the other, and head in opposite directions (one car heads directly North and the other car heads directly South). The North bound car has a speed of 35 miles per hour. The speed of the South bound car is unknown, call this speed v miles per hour. (Assume v is a positive number).

(a) What is the function, $d(t)$, that gives the distance between the cars in miles, when t is in hours? Your answer will involve the unknown parameter v.



N at $(0, 35t)$

S at $(3, -vt)$

$$d(t) = \sqrt{3^2 + (35t + vt)^2}$$
$$= \sqrt{9 + (35+v)^2 t^2}$$

(b) If 42 minutes later, the cars are 50 miles apart, how fast is the South bound car going?

$t = \frac{42}{60}$ $\quad d = 50$

$$50 = \sqrt{9 + (35+v)^2 \left(\frac{42}{60}\right)^2}$$
$$2500 - 9 = (35+v)^2 \left(\frac{42}{60}\right)^2$$
$$5083.67 = (35+v)^2$$
$$71.300 = 35 + v \qquad v = 36.3 \text{ miles per hour}$$