

Names: \_\_\_\_\_

## Exponential Modeling, via the UN

Due Wednesday, Dec. 4th.

Please write down the data that you were supposed to collect last week, so I can use it for grading.

**Country from Category 1:** \_\_\_\_\_

Raw Pop. in: 1950 \_\_\_\_\_, 2000 \_\_\_\_\_, 2015 \_\_\_\_\_,  
2025 \_\_\_\_\_, 2050 \_\_\_\_\_.

Annual Pop. Growth Rate for: 1995-2000 \_\_\_\_\_, 2000-05 \_\_\_\_\_,  
2010-15 \_\_\_\_\_, 2020-25 \_\_\_\_\_, 2045-50 \_\_\_\_\_.

**Country from Category 2:** \_\_\_\_\_

Raw Pop. in: 1950 \_\_\_\_\_, 2000 \_\_\_\_\_, 2015 \_\_\_\_\_,  
2025 \_\_\_\_\_, 2050 \_\_\_\_\_.

Annual Pop. Growth Rate for: 1995-2000 \_\_\_\_\_, 2000-05 \_\_\_\_\_,  
.2010-15 \_\_\_\_\_, 2020-25 \_\_\_\_\_, 2045-50 \_\_\_\_\_.

1. Last week we looked at a particular model of population growth. Today we'll examine how that model compares to the data you collected.

(a) Using the raw population numbers (not the rates) from 1950 and 2000, which are actual numbers (not estimated), form an exponential model  $P_1(t)$  of the population of Country 1. Set  $t = 0$  at the year 2000 (So 1950 is  $t = -50$ .)

(b) Using the raw population numbers from 1950 and 2000, form an exponential model  $P_2(t)$ . Again, set  $t = 0$  at 2000.

(c) Fill in the 1st, 2nd, 4th and 5th rows in the table below. We'll fill in the rest in later.

Pop. of Country	1950	2000	2015	2025	2050
Ctry1 ( $P_1(t)$ )					
Ctry1 (U.N. est.)					
Ctry1 (Problem 2)					
Ctry2 ( $P_2(t)$ )					
Ctry2 (U.N. est.)					
Ctry2 (Problem 2)					

(d) (Note that the 2015, 2025, 2050 numbers that are given are not necessarily what will be, because nobody's been to 2015 yet. The U.N. made these with their own mathematical models.) Do your numbers match the U.N. estimate closely? What does that tell you about your model?

2. Let's look at the growth rates now. Recall that our model from last week was

$P(t) = P_0(1+r)^t$ , where  $P_0$  is the population at time  $t = 0$ ,  $t$  is in years, and  $r$  is our annual growth rate. (Here, it's an overall growth rate; the birth and death rates are already factored in.)

(a) What is your annual growth rate,  $r$ , for  $P_1$ ? For  $P_2$ ? (Your answer should be a decimal less than one. If you multiply the rate by 100, you get a percent rate, which is the units the UN data is given in.)

(b) Notice that the rates you used for  $P_1$  and  $P_2$  are constant. The rates from the U.N. website change from year to year. Look at the growth rates for Country 1 (which should be the lesser-developed country). What trend do these numbers follow? Are they increasing, decreasing, or both? Why would these numbers act like this? Remember this is the *rate* of growth, not the amount of growth itself.

(c) Look at the growth rates for Country 2 (the more developed country). What trend are these numbers following? Are they increasing, decreasing, or both? What do the negative numbers mean?

(d) We can't make a straight up exponential model with the U.N.'s rates, because the rates are changing. But we can do some computations to guess what the population will be (even though we already have the U.N.'s estimates.) Call the U.N. yearly growth rate for the years 2000-2005  $r_1$ . Multiply the U.N. 2000 population by  $(1+r_1)^{10}$  to find an estimate of the population in the year 2010. (Don't forget to divide the UN rate by 100 to go from a percent value to a decimal value.)

(e) Call the U.N. growth rate for Country 1 for the years 2010-2015,  $r_2$ . Get an estimate for the population of Country 1 for the year 2015 by multiplying your answer from part (d) by  $(1+r_2)^5$ . The 5 corresponds to the 5 years between 2010 and 2015. Fill this number in the table.

(f) Call the U.N. growth rate for Country 1 for the years 2020-2025,  $r_3$ . Get an estimate for the population of Country 1 for the year 2025 by multiplying your answer from part (e) by  $(1+r_3)^{10}$ . The 10 corresponds to the 10 years between 2015 and 2025. Fill this number in the table.

(g) Call the U.N. growth rate for Country 1 for the years 2045-2050,  $r_4$ . Get an estimate for the population of Country 1 for the year 2050 by multiplying your answer from part (f) by  $(1+r_4)^{25}$ . The 25 corresponds to the 25 years between 2025 and 2050. Fill this number in the table.

(h) Notice that to get the numbers we used what was essentially an exponential model. The only difference is that we changed the rate (and hence the base  $b$ ) every few years. Do these numbers match up better to the U.N. estimates than your model from problem 1? (They actually might not match up better. That's ok.)

(One more problem on the back.)

(i) Repeat what you did in steps (e) through (g) to fill in the last row on the table with an estimated population for Country 2.