Names:____________________

# Multiplying like Rabbits

This worksheet is due next Wednesday. Bring it to class Monday for reference.
**NOTE:** For next Monday, you need to pick two countries, one from each of the lists below. Go to http://www.un.org/esa/population/unpop.htm and find the real or estimated populations and annual population growth rates for these countries. The information is in the World Population section. There's a lot of interesting information there. (C1: Afghanistan, Iraq, Jamaica, Mali, Mongolia, Tunisia C2: Belgium, Germany, Italy, Japan, Portugal, Slovakia)

**Country 1**:___________________

Pop. in: 1950______________, 2000______________, 2015______________,

2025______________, 2050______________.

Annual Pop. Growth Rate for: 1995-2000______________, 2000-05______________,

2010-15______________, 2020-25______________, 2045-50______________.

**Country 2**:___________________

Pop. in: 1950______________, 2000______________, 2015______________,

2025______________, 2050______________.

Annual Pop. Growth Rate for: 1995-2000______________, 2000-05______________,

2010-15______________, 2020-25______________, 2045-50______________.

1.(a) Imagine a colony of bacteria. Each bacterium undergoes mitosis (splits itself in half) once a day. In other words, if there's one bacterium on day 0, on day 1 there are 2 bacteria. Let t be the number of days, and let B(t) be the number of bacteria at the end of t days. Complete the table to the right.

| **t** | **B(t)** |
|---|---|
| 0 | 1 |
| 1 | 2 |
| 2 | 4 |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |

(b) Find an exponential formula for B(t).

2. (a) Now assume that at t = 0, we have 10 bacteria. If the growth rate is the same, complete the table to the right.

| **t** | **B(t)** |
|---|---|
| 0 | 10 |
| 1 | 20 |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |

(b) Find an exponential formula for B(t) in this case.

(c) Assume we start with C bacteria at t = 0. How many bacteria will we have at the end of t days? Find the formula.

3. (a) Now assume that we just have one bacterium at t = 0, but the little buggers undergo mitosis twice a day. Fill in the table.

| **t** | **B(t)** |
|---|---|
| 0 | 1 |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | 1,024 |
| 6 | |
| 7 | |

(b) Find an exponential formula for B(t) in this case. Write it as "2 to the something".

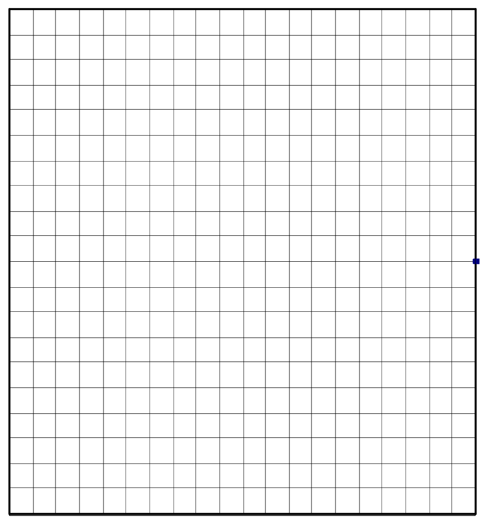
(c) Assume that the bacteria multiply n times a day. How many bacteria will we have after t days? Find the formula. You might want to check your formula by picking n =3 and t =1, and seeing if it makes sense.

4. Combine the ideas from the above problems to write out a general formula B(t) for the number of bacteria we'll have at the end of day t, if we start with C bacteria at t = 0, and they split n times a day.

5. People are not bacteria, but we can describe their growth rate in similar ways. Though we don't undergo mitosis, we can say, for example, that, on average, every year one in 20 people will have a baby. So for every 20 people in year 0, there are 21 in year 1. This would mean that, if the population started at 1,000 people, after 1 year there would be $1{,}000\left(1+\frac{1}{20}\right)=1000\left(\frac{21}{20}\right)=1{,}050$ people. After 2 years, there would be $\left[1{,}000\left(\frac{21}{20}\right)\right]\left(\frac{21}{20}\right)=1{,}000\left(\frac{21}{20}\right)^2$ people, and so forth. In this case 1/20 is called the birth rate.

(a) If at time t = 0, the population was $P_0$, find the number of people after t years, using the ideas in problems 1-4. Call this function B(t).

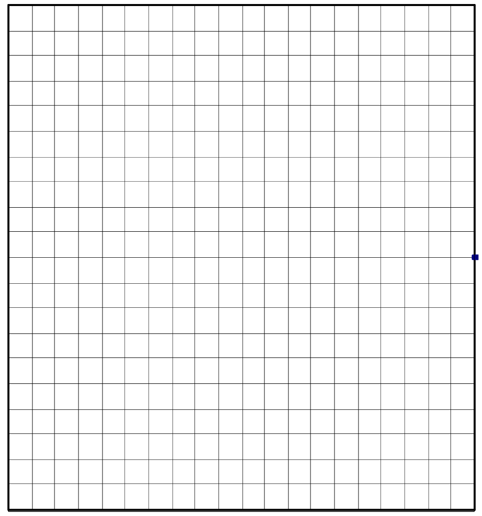
(b) Suppose $P_0$=1,000. Graph B(t) from t = 0 to t = 50. Notice what happens as t gets big. This is because the yearly multiplier, 21/20 is greater than one.



6. The model in problem 5 assumes that no one ever dies. For the purposes of this problem, let's assume no one is ever born. Assume one person out of 25 dies every year. Then for every 25 people there are in year 0, there will only be 24 in year 1. So if the population started at 1,000, after 1 year there would be $1{,}000\left(1-\frac{1}{25}\right)=1{,}000\left(\frac{24}{25}\right)=960$ people. After 2 years there would be $1{,}000\left(\frac{24}{25}\right)^2$ people. Here 1/25 is called the death rate.

(a) If at time t = 0, the population was $P_0$, how many people would there be after t years, assuming the above growth rate? Call this function D(t).

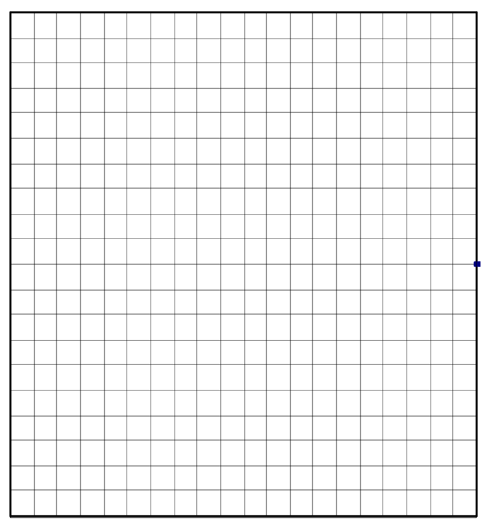
(b) Suppose $P_0$=1,000. Graph D(t) from t = 0 to t = 50. Notice what happens as t gets big. This is because the yearly multiplier, 24/25 is less than one.



7. Now let's combine the two above models. Assume we have an initial population of 1,000 people.

(a)Assume that, every year, 1 out of 20 people who were in the population at the beginning of the year have a baby. AND assume that every year 1 out of 25 people who were alive at the beginning of the year die. At the end of year 1, then, we will have $1{,}000+1{,}000\left(\frac{1}{20}\right)-1{,}000\left(\frac{1}{25}\right)=1{,}000\left(1+\frac{1}{20}-\frac{1}{25}\right)=1{,}000\left(\frac{101}{100}\right)=1010$ people. Notice what we did: The birth rate multiplier was $1+\frac{1}{20}$ = 1 + the birth rate. The death rate multiplier was $1-\frac{1}{25}$ =1 – the death rate. We combined the two to get a total multiplier of 1+birth rate-death rate. We'll call (birth rate – death rate) = growth rate. Find a formula P(t) for the population after t years, using this model.

(b) Graph P(t) from t = 0 to t = 50. Notice what happens as t gets very large. Why is this? What are the ramifications for the world if this population model is accurate?



(c) What would the graph look like if the death rate was greater than the birth rate? For example, what if the death rate was 1/20, while the birth rate was 1/25? If you don't instantly see what the graph would look like, try actually plotting the new P(t) using these numbers. Regardless of the actual birth and death rates, note that this model is always exponential. On next week's worksheet, we'll check these models according to actual statistical data.