How Much Soap Can You Buy with Two Million Dollars?

You’re in your new job as sales analyst for VariKleen Soap, Inc, when your bosses tell you, “We need a profit analysis on our new line of Very VariKleen soap.” You have the following data: If you charge $1 per bar of soap, you’ll sell 2 million bars. If you charge $2 per bar, you’ll sell 1.5 million bars of soap.

1. Make a linear model p(q) of the price, p, you should sell the soap at if you want to sell q million bars. (Note that q will be in millions of bars. You’ll have values like q=2 or q=1.5 and NOT values like q=2,000,000 or q=1,500.00.

2. Your revenue, R, is the total amount of money you take in, in this case from selling soap. It’s given by the formula R=qp, where q is the number of items sold and p is the price you sell them at. Think about why this makes sense, and then plug your answer from part 1 into this formula to get rid of the p in the above formula. You should get R(q), a function of only q. This function tells you how much money you’ll bring in if you sell q million items.

3. Find the values of q where your revenue is zero. Do these make sense? Graph R(q) on the graph on the next page with q along the x-axis and millions of dollars along the y-axis.
4. Your profit, P, is your revenue minus expenses. It’s given by the formula P=R-C, where C are the costs you incur. 

**Example:** You make $400 a week at your job, but have to pay $350 in bills every week. Your revenue is R=400. Your costs are C=350. Your profit (how much you’ve made) is \( P = R - C = 400 - 350 = 50 \) dollars.

In the case of VariKleen Soap, the costs you incur are due to production, shipping, advertising, etc. You know that your costs are given by the function

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C(q) = 15q^2 - 13q + 15.
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GOES NEGATIVE!!!!! YUCK!!! Find the formula P(q) for your profit. (Be careful not to get the price, p, mixed up with the Profit, P.)

5. Using the fact that C(q) is a parabola, graph C(q) on the same graph as R.

6. Using the vertex formula, find the q where you attain your maximum profit. Draw the vertical line on the graph that corresponds to this value of q. If you drew R(q) and C(q) right, this should be a special point in some way. How is it special? Can you say why the maximum profit is at the q value where this happens?