MATH 120 U

EXAM 1

AUTUMN 2002

Your Name

Problem	Total Points	Score
1	8	
2	6	
3	12	
4	12	
5	12	
6	10	
Total	60	

- Don't cheat.
- Don't stress out.
- Calculators are not allowed on this exam.
- You may use one double-sided handwritten page of notes.
- In order to receive credit, you must show your work.
- Place a box around your final answer to each question.
- If you need more room, use the backs of the pages and indicate where to look.
- Raise your hand if you have a question.

1 (8 points) Let $f(x) = 3x^2$, $g(x) = x - 2$, and $h(x) = 7$. Find and simplify:

(a) $f(g(x))$

(c) $f(h(x))$

(b) $g(f(y))$

(d) $\frac{g(3)}{h(3)}$

2 (6 points) Let $f(x) = 3x^2$, and let $u(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } 0 \le x < 1 \\ 0 & \text{if } x \ge 1 \end{cases}$.

Graph $f(x+3)u(2x+1)$.

3 (12 points) Uncle Norm and his wife, Aunt Norma, go camping in the mountains. While they're making their breakfast, a meteorite crashes into the ground 2 miles west and 6 miles north of their campsite. It makes a circular crater in the ground of radius 2 miles.

(a) If their campsite is at the origin, find an equation for the edge of the crater.

(b) Uncle Norm and Aunt Norma, always the curious pair, start jogging in a straight line from their campsite towards the crater, until they reach the edge. They jog two miles north for every mile they travel west. Find an equation of the line that their path lies on.

(c) If they jog at 5 miles/hour, how long does it take them to reach the edge of the crater?

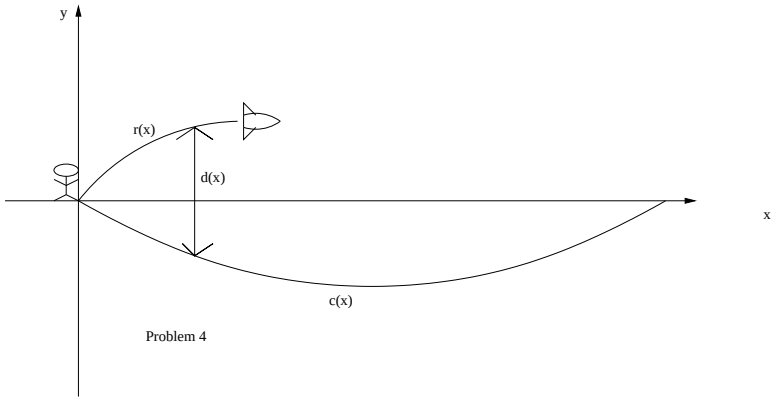
4 (12 points) **(When working on this problem, ignore all data from the previous problem.)** For this problem, refer to the picture on the next page. You can rip out that page if you like.

When they reach the edge of the crater (which is 4 miles in diameter), Aunt Norma pulls out her surveillance rocket, which she always carries with her. She shoots the rocket from the edge of the crater. It flies in a parabola, and hits its maximum height of $\frac{1}{4}$ mile high after its flown 1 mile horizontally. (For the purposes of this problem, don't worry about multipart functions, step functions, or whether certain domains are physically valid.)

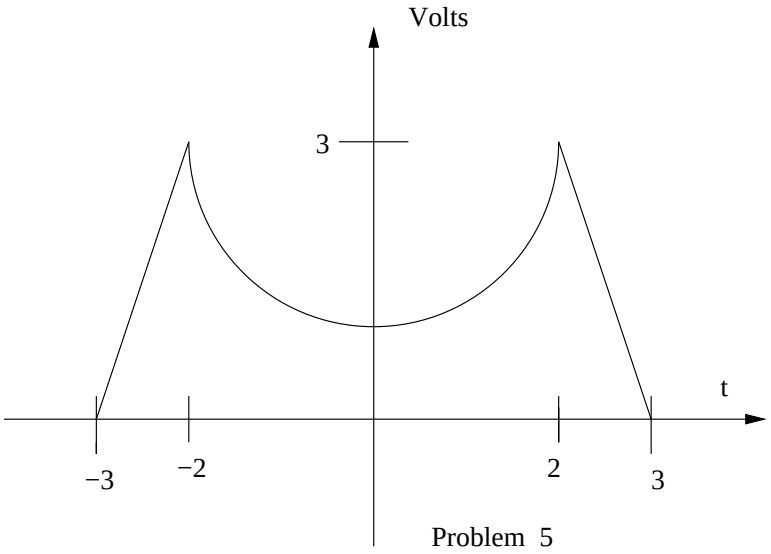
(a) Find a function $r(x)$ for the rocket's height as a function of the horizontal distance it has travelled. Put the origin at the edge of the crater where Aunt Norma is standing.

(b) The rocket measures the vertical distance $d(x)$ between the rocket and the bottom of the crater. It gives the formula $d(x) = -\frac{3}{8}x^2 + x$. Assuming the crater has a parabolic shape, find a formula $c(x)$ for the depth of the crater as a function of the distance from the edge. (Hint: Be very careful what you add or subtract from what. The height of the crater should be negative, at, for example x=1. Check some points to make sure your c(x) makes sense.)

(c) At what x-value is the deepest part of the crater? How deep is it?



Problem 4



Problem 5

5 (12 points) **(When working on this problem, ignore all data from the previous problems.)** For this problem, refer to the figure on the previous page. You can rip out that page if you like.

Uncle Norm and Aunt Norma climb down the crater. When they get to the bottom, the meteorite turns out to be a spaceship. An alien pops out and says, "Oh no! My spaceship is busted. What shall I do?" Uncle Norm pulls out his handy voltmeter to take a look at the spaceship. As output he gets the graph on the previous page, which is two line segments and a half-circle.

(a) Write out a multipart function $V(t)$ that gives the graph.

(b) The alien says, "Oh no, that's not how it's supposed to look! For my ship to be able to fly, the graph needs to start two seconds later (shifted to the right), and it needs to be twice as wide (dilated horizontally)." Give the equation for such a function. (Hint: First write out how you would change an arbitrary V(t), then write what that means in terms of the specific function you have above.)

6 (10 points) **(When working on this problem, ignore all data from the previous problems.)** Uncle Norm fixes the alien's spaceship. The alien says, "Oh thank you! You humans are so helpful (and very good with math!) My name is Glbl. I was wondering if you could help me one more time. I left my home on Betelgeuse 3 at 3:00am this morning, and travelled toward Hypoxion 7 at a rate of 20 million miles per hour. My friend Zzx'dx left Hypoxion 7 at 4:00am and travelled toward Betelgeuse 3. She said she was going to arrive at Betelgeuse 3 at 9:00am. Betelgeuse 3 and Hypoxion 7 are 150 million miles from each other. Unfortunately, we ran into each other in passing, which is how I got stranded here. Can you tell me what time we hit each other and where? I'm rather worried about her. She *does* fly rather fast. Everyone knows the speed limit is 20 million miles per hour. Beyond that it's just not safe."

Find where and when Glbl hit Zzx'dx. (Hint: First find linear functions $g(t)$ and $z(t)$ for Glbl and Zzx'dx's distance from Betelgeuse 3. Put your units in millions of miles.)