Math 120 Final Examination Autumn 2002

Your Name

Your Signature

Student ID #

Quiz Section

Professor’s Name

TA’s Name

• This exam is closed book.

• You may use one $8\frac{1}{2} \times 11$ sheet of notes. Students may not share notes.

• Calculators are allowed. Students may not share calculators.

• In order to receive credit, you must show your work. Do not do computations in your head. Instead, write them out on the exam paper.

• If you use a trial and error (or guess and check) method when an algebraic method is available, you will not receive full credit.

• If you need more room, use the backs of the pages and indicate to the reader that you have done so.

• Raise your hand if you have a question.

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1. (16 points)

(a) (4 points) Solve for $x$:

\[
\frac{x^2 - 3x + 4}{x + 1} = 2x - 1.
\]

(b) (4 points) Let $f(x) = \frac{1}{x + 4}$. Compute $\frac{f(x + h) - f(x)}{h}$ and simplify as much as possible.
(c) (4 points) Let \( g(x) = \frac{x + 3}{2x - 5} \). Compute \( g^{-1}(x) \).

(d) (4 points) Find the zeros and asymptotes, if any, of the following function:

\[
h(x) = \frac{x^2 - 1}{x^4 + 4}
\]
2. (12 points) Let \( f(x) = \begin{cases} \sqrt{9-x^2} & \text{if } -3 \leq x \leq 0 \\ 3 - 2x & \text{if } 0 \leq x \leq 3. \end{cases} \)

The graph of \( f(x) \) is given at right.

(a) (4 points) Compute the domain of \( g(x) = f\left(\frac{1}{2}x - 6\right) \).

(b) (4 points) Compute the domain of \( h(x) = \sqrt{f(x)} \).

(c) (4 points) The graph below is the graph of \( k(x) = af(x - b) \), for some real numbers \( a \) and \( b \). Determine the values of \( a \) and \( b \).
3. (15 points) Clyde is selling tickets to the Math Olympiad. The price he charges per ticket depends on the number of tickets you buy. If you buy 4 tickets from him, he charges $15 each. If you buy 8 tickets, he charges $12 each.

(a) (4 points) Give a linear function $P(x)$ that computes the price per ticket if you buy $x$ tickets.

(b) (4 points) Clyde’s revenue from a sale is the total amount he charges for the entire order. What is his revenue for an order of 14 tickets?

(c) (3 points) Give a (quadratic) function $R(x)$ that computes Clyde’s revenue from selling $x$ tickets.

(d) (4 points) How many tickets should he sell to maximize his revenue?
4. (15 points) A snail is moving along the top of a carpenter’s worktable. The top of the worktable is a 100 centimeter by 100 centimeter square. The snail starts at one corner and moves in a straight line toward the midpoint of one side as pictured below. There is a circular piece of sandpaper on the table, with its center 40 cm from the side labeled \(A\) and 17 cm from the side where the snail is heading. The piece of sandpaper has radius 10 cm. If the snail always moves 0.3 cm/sec, how much time (in seconds) does the snail spend crawling over the sandpaper?
5. (14 points) Squirrel is baking a cake using an experimental recipe that results in the cake rising and falling while baking. When the cake is placed in the oven its height begins to decrease. Once it reaches its minimum height of 3.6 cm, the height increases, reaching a maximum of 7.8 cm after 19 minutes in the oven. After 35 minutes in the oven, the height of the cake is at a minimum for the second time.

Suppose the height of the cake is a sinusoidal function of time.

(a) (8 points) Find a sinusoidal function \( h(t) \) that represents the height of the cake after \( t \) minutes in the oven.

(b) (6 points) Squirrel decides to take the cake out when it reaches a height of 7 cm just before it would attain its maximum height for the third time. How long is the cake in the oven?
6. (15 points) A farming town in Peru grew 84,000 potatoes in 1994. The population of the town that year was 600. In 1998 the population was 632. In 2000 they grew 96,430 potatoes.

(a) (5 points) Give an exponential function in the form \( y = Ae^{rt} \) relating the population \( y \) of the town to the year \( t \). (Take \( t = 0 \) in 1994).

(b) (5 points) Give an exponential function in the form \( z = Ae^{rt} \) relating the number of potatoes \( z \) to the year \( t \).

(c) (5 points) In what year will they grow 200 potatoes per person in the town?
7. (13 points) Anna is on a merry-go-round with radius 9 feet. The merry-go-round spins counterclockwise so that Anna’s linear speed is 12 feet per second. At time $t = 0$, Billy is 13 feet east and 10 feet north of the center of the merry-go-round and Anna is directly south of the center, as pictured.

(a) (5 points) Give Anna’s $x$- and $y$-coordinates at time $t$ seconds.

(b) (8 points) At time $t = 0$, Billy begins to run at a constant speed in a straight line toward the point $P$. At $t = 7$ seconds, Billy reaches the spinning merry-go-round and jumps on at the point $P$, directly east of the center. Give the multi-part rules for Billy’s $x$- and $y$-coordinates at time $t$ seconds, for $t \geq 0$. 