These problems are a selection of problems of the type you can expect to find on the final exam. (They are, for the most part, lifted from old exams from various Math 120 instructors.) This selection should not be considered a sample exam, since the exam will be written so that it should take two hours. Answers will be available on the class web page.

**Important things to remember** about the final exam: It is Saturday, December 15th, from 5:00 PM to 8:00 PM, in a room that is not the usual lecture hall. Also, you may use a scientific calculator, but you not use a graphing calculator, nor may you share calculators. (PDAs, such as Palm Pilots, also may not be used as calculators.)

1. Captain Archer has taken the Enterprise to the star system Sirius. The star has a radius of 1,000,000 kilometers. He sees a particularly interesting sunspot on the star that he wishes to study and orders the helm to put the ship in an orbit 5,000,000 km above the surface of the star and to keep the ship directly over the sunspot while it is observed.

   (a) If Sirius rotates once in twenty days, what is the linear velocity of the Enterprise (in kilometers per minute, please)?

   (b) After gathering sufficient data on the sunspot, Archer has the ship put into an orbit that takes it around the star moving at a linear velocity of 20,000 kilometers per minute. Suddenly they see a flare thirty degrees ahead of them. Assume that the flare moves with the star. SHIELDS UP!

      (i) When does the Enterprise run into the flare? (Keep in mind that the star and the flare are rotating together as the Enterprise rotates about the star.)

      (ii) Where is the Enterprise when it runs into the flare? (What is its angle?)

2. Sketch the function $f(x) = \frac{2x^2 - 8}{x^2 - 1}$. Be sure to carefully label your graph ($x$-intercept, $y$-intercept, vertical asymptotes, and horizontal asymptotes).

3. The initial weight of a starving animal is $W_0$. Its weight after $t$ days is given by

   $$W(t) = W_0 e^{-0.007t}.$$

   (a) What percentage of its weight does it lose in the first day?

   (b) After how many days will the animal weigh 80% of its initial weight?

4. If $f(x) = \frac{1 - 2x}{3x + 1}$ and $g(x) = \cos \left( \frac{3 + 2x}{2 + 3x} \right)$, find $f(g(x))$ and $g(f(x))$. 
5 Suppose \( \cos^{-1}(\cos(\theta)) = \theta \), but \( \sin^{-1}(\sin(\theta)) \neq \theta \). In which quadrant is the point \((x, y) = (\cos(\theta), \sin(\theta))\)? (You may assume that the point \((x, y)\) does not lie on either coordinate axis.)

6 Find all solutions \( x \) to the equation \( 2^{x^2+x} = 8 \). Leave your answers in exact form (no decimals).

7 A certain lake is stocked with fish. A standard model of population growth in an area of limited food resources leads to the following formula for the population \( N \) of fish in the lake \( t \) weeks after the fish are introduced to the lake:

\[
N = \frac{750}{2 + 3e^{-(0.0896)t}}.
\]

(a) What is the number of fish initially?

(b) After how many weeks has the number of fish doubled?

(c) After a long time has passed, the population essentially stops growing and remains constant. What is the size of this steady-state population? (That is, what does \( N \) become when \( t \) becomes very large?)

8 Mark is disgruntled with being a TA and not an instructor. To vent his frustration, he builds a large catapult and is currently trying to hurl a large rock. As described in the picture, the catapult will launch the rock with a horizontal speed \( v_x = 22.5 \) miles per hour, and a vertical speed of \( v_y = 30 \) miles per hour. The launching point is 20 feet above the ground, at the coordinates \((0, 20)\) in the coordinate system shown.

(a) Find the initial speed of the rock.

(b) Find the angle \( \theta \) in the picture.

(c) The rock follows the parametric equations

\[
\begin{align*}
x(t) &= 22.5t + A \\
y(t) &= -16t^2 + 30.0t + B.
\end{align*}
\]

Find \( A \) and \( B \).

(d) Find the equation that describes the path of the rock. That is, find \( y \) in terms of \( x \) for the rock.

(e) When does the rock reach its maximum height? Give the rock’s \((x, y)\) coordinates when it is at this height.
Lou borrows $1,000 from his credit card, which charges 18% interest compounded monthly.

(a) Find a formula for $L(t)$, the amount Lou owes after $t$ years. Only consider the original $1,000 and accumulated interest.
(b) With $200 of his new-found wealth, Lou pays off some bills. The remaining $800 he invests in a freelance precalculus company that he believes will earn 20% interest compounded continuously. Find the value $I(t)$ of Lou’s investment after $t$ years.
(c) Lou’s net worth after $t$ years is $N(t)$, the difference of his investment value $I(t)$ and his accumulated loan balance $L(t)$. That is, $N(t) = I(t) - L(t)$. Find Lou’s net worth after 5 years.
(d) When does Lou break even? That is, when is Lou’s net worth zero?

Particle A starts at the point $(1, 2)$ and moves in a straight line toward the point $(-5, 10)$ at a constant speed of 5 units/sec. Particle B starts at the point $(0, 2)$ and moves in a straight line at a constant speed toward the point $(-4, 0)$. It takes Particle B two seconds to move from $(0, 2)$ to $(-4, 0)$. Both particles start at the same time and move forever. When are they 7 units apart?

Anne is building a bicycle. She can pedal a constant 80 revolutions per minute, and she wants the bicycle to go a steady 20 miles per hour when she does. The rear wheel is 28 inches in diameter and the rear sprocket has a radius of 2 inches. What should Anne make the radius of the front sprocket so she can ride at 20 mph?

A right triangle is placed in the corner of the first quadrant of a coordinate system. A rectangle fits inside the triangle with two sides of the rectangle running along the two legs of the right triangle, and the vertex opposite those two sides lying on the hypotenuse. (See the picture.)

(a) Find a formula for the width $w$ of the rectangle as a function of the height $h$.
(b) Find a formula for the area $A$ of the rectangle as a function of its height.
(c) What is the largest rectangle (in terms of area) that will fit in the triangle in this way?
13 (a) Sketch the graph of \( y = 5 - \ln[2(x - 3)] \). Indicate the following points, if they occur on your graph: where the curve crosses each axis, where \( x = 5 \), and where \( y = 5 \).

(b) Find the domain and range of the function \( y = 5 - \ln[2(x - 3)] \) that you graphed in part (a).

14 You are studying the winter temperature in northern Montana. Over two and a half days (60 hours), the temperature varies sinusoidally, as shown in the graph, below.

The high temperature is 27\(^\circ\) Fahrenheit at 1:00 PM each day, and the low temperature of \(-3\)\(^\circ\) Fahrenheit occurs at 1:00 AM. The graph shows temperature versus time, in hours, since midnight on January 1.

(a) Find the amplitude, the period, the mean temperature, and a phase shift. Label the points (with coordinates) on the graph corresponding to points that are maxima, means (on the mean line), or minima.

(b) Find the formula 

\[ T(t) = A \sin \left( \frac{2\pi}{B} (t - C) \right) + D \]

for temperature \( T \) (in degrees Fahrenheit) in terms of time \( t \) (in hours since midnight, January 1).

(c) We’d like to have a picnic lunch at noon or tea at 3:00 PM. What are the temperatures at these times?

(d) Find how many hours (during the 60 hours shown on the graph) the temperature was below 0\(^\circ\) Fahrenheit.