1. (8 points) Differentiate the following functions. BOX your final answer. No need to simplify.

   (a) \( y = x \ln(x) + 10e^{-x^2} \)

   \[
   \frac{dy}{dx} = (x) \ln(x) + x (\ln(x))^2 + 10 e^{-x^2} \left( -2x \right)
   \]

   \[
   = \ln(x) + x \cdot \frac{1}{x} + 10 e^{-x^2} (1-5x^2)
   \]

   (b) \( g(t) = \left[ 1 + (\ln(t))^2 \right]^{\frac{1}{5}} \)

   \[
   g'(t) = \frac{1}{5} \left[ 1 + (\ln(t))^2 \right]^{-\frac{4}{5}} \cdot 2 (\ln(t)) \cdot \frac{1}{t}
   \]

2. (8 points) Based on the graph of \( f(x) \) shown below, identify which of the points A through G marked on the graph satisfy each of the following conditions.

   (List all that apply, no need to justify your answers.)

   - Critical point(s) for \( f(x) \): \( B, D, F \) ← point where \( f'(x) = 0 \) so where \( \tan \) line is horizontal
   - Inflection point(s) for \( f(x) \): \( C, E, F \) ← point where concavity changes
   - Point(s) where \( f'(x) > 0 \) and \( f''(x) > 0 \): \( G \) ← point where \( f(x) \) is ↑ and concave up
   - Point(s) where \( f'(x) = 0 \) and \( f''(x) = 0 \): \( F \) ← horizontal inflection point
3. (8 points) Evaluate the following integrals. Simplify and BOX your final answer.

(a) \[ \int \left( \frac{3}{x} - \frac{5}{\sqrt{x}} + 2e^{5x} \right) \, dx = \int \frac{3}{x} \, dx - 5 \int x^{-\frac{1}{2}} \, dx + 2 \int e^{5x} \, dx \]

\[ = 3 \ln(x) - 5 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + 2 \frac{e^{5x}}{5} + C \]

\[ = \boxed{3 \ln(x) - 10 \sqrt{x} + \frac{2}{5} e^{5x} + C} \]

(b) \[ \int_{1}^{2} 12(t - 3)^2 \, dt = \int_{1}^{2} 12 \left( t^2 - 6t + 9 \right) \, dt \]

\[ = \int_{1}^{2} 12t^2 - 72t + 108 \, dt \]

\[ = \left. \left( 12 \frac{t^3}{3} - 72 \frac{t^2}{2} + 108t \right) \right|_{1}^{2} \]

\[ = \left( 4t^3 - 36t^2 + 108t \right) \bigg|_{1}^{2} \]

\[ = (4(2)^3 - 36(2)^2 + 108(2)) - (4(1)^3 - 36(1)^2 + 108(1)) \]

\[ = 104 - 76 \]

\[ = \boxed{28} \]
4. (8 points) A company is selling Items. The marginal revenue, in $/Item, at q hundred Items is 
\( MR(q) = 29 - 2q \), the marginal cost is \( MC(q) = 5 \), and the fixed costs are \( FC = 3 \) hundred $.

(a) What number of items results in a maximal profit?

\[
MR(q) = MC(q) : \quad 29 - 2q = 5 \\
24 = 2q \\
q = 12
\]

\( \text{ANSWER: } q = 12 \) hundred Items

(b) What is the maximum profit?

\[
TR(q) = 29q - q^2 \\
TC(q) = 5q + 3
\]

\[
\Rightarrow P(q) = -q^2 + 24q - 3 \\
P(12) = -12^2 + 24(12) - 3
\]

\( \text{ANSWER: } 141 \) hundred dollars.

5. (6 points) The graph of a function \( f(x) \) is shown below.

(a) Compute the value of the following integral.

\[
\int_0^9 f(x)dx = \left[ -1 \right]
\]

\[
= \frac{3}{3} + \frac{7}{1} \\
= -\frac{1}{2}(3)(3) + \frac{1}{2}(1)(2) \\
= -\frac{9}{2} + \frac{2}{2} = -2
\]

(b) Define a new function \( A(m) = \int_0^m f(x)dx \), where \( f(x) \) is the function in the graph above.

i. Compute \( A(1) = 8.5 \)

ii. At what value(s) of \( m \) does \( A(m) \) have a local minimum? \( m = 8 \)

iii. At what value of \( m \) in the interval from \( m = 0 \) to \( m = 10 \) is \( A(m) \) largest? \( m = 10 \).
6. (12 points) You run a business selling lollipops. Your profit, in dollars, from selling $q$ thousand lollipops is given by the function:

$$P(q) = q^4 - 32q^3 + 270q^2 - 200.$$

Use the methods of this class and show all your work.

(a) Find all the critical values of the profit function.

\[
\begin{align*}
P'(q) &= 4q^3 - 96q^2 + 540q = 0 \\
&= 2(2q^3 - 48q^2 + 270q) = 0 \\
&\downarrow \text{ divide by } 2 \\
q &= 0 \text{ or } q^2 - 24q + 135 = 0 \\
q &= \frac{24 \pm \sqrt{36}}{2} = \frac{24 \pm 6}{2} \\
&= 15, 9
\end{align*}
\]

\textbf{ANSWER: (list all)} $q = 0, 9, 15$ thousand lollipops.

(b) Find the minimum and the maximum profit you can make if you sell between 1 thousand and 10 thousand lollipops.

\textbf{Closed Interval Method: min/max at CV or endpoints}

\[
\begin{align*}
P(1) &= 1^4 - 32(1)^3 + 270(1)^2 - 200 = 39 \\
P(9) &= 9^4 - 32(9)^3 + 270(9)^2 - 200 = 4903 \\
P(10) &= \ldots = 4800
\end{align*}
\]

\textbf{ANSWER:} Min Profit is $\$39$ at $q = 1$ thousand lollipops.

Max Profit is $\$4903$ at $q = 9$ thousand lollipops.

(c) Find the longest interval on which the profit function is concave down.

$$P''(q)$$ is concave down when $P''(q)$ is negative (below x-axis)

\[
\begin{align*}
P''(q) &= 12q^2 - 192q + 540 = 0 \\
&\downarrow \text{ divide by } 12 \\
q^2 - 16q + 45 = 0 \\
&\quad \text{divides by } 2 \\
&\quad q = \frac{16 \pm \sqrt{16^2 - 4 \cdot 45}}{2} \\
&\quad q = \frac{16 \pm 8.747\ldots}{2}
\end{align*}
\]

\textbf{ANSWER: From } $q = 3.64$ to $q = 12.36$ thousand lollipops