Math 112, Spring 2017, Solutions to Midterm II

1. (a) $\dfrac{d}{dx}\ln\left(3x^2+3x+5\right) = \dfrac{6x+3}{3x^2+3x+5}$

   (b) $\displaystyle\int \left(5x^3-\sqrt{x}+e^{2x}-8\right)\,dx = \frac{5}{4}x^4-\frac{2}{3}x^{3/2}+0.5e^{2x}-8x+C$

   (c) $\displaystyle\int_1^2 \frac{x^2+1}{x^2}\,dx = \int_1^2 1+x^{-2}\,dx = x-x^{-1}\Big|_1^2 = \left(2-\frac{1}{2}\right)-(1-1) = \frac{3}{2}$

2. (a) Where $MR''=0$:

$$MR'(x) = 0.003x^2-0.24x+3.072$$
$$MR''(x) = 0.006x-0.24 = 0$$

   so $x = 0.24/0.006 = 40$.

   (b) Where $MC'(x)=0$:

$$MC'(x) = 0.003x^2+0.24x-4 = 0$$
$$x = \frac{-0.24\pm\sqrt{0.24^2-4(0.003)(-4)}}{0.006} \approx 14.16 \text{ (the other root is negative)}$$

   So the minimum value is $MC(14.16) = 50.26$.

   (c) $\displaystyle\int_{10}^{45} 0.001x^3+0.12x^2-4x+80\,dx$

   (d) Profit is maximized when $MR=MC$:

$$0.001x^3-0.12x^2+3.072x+200 = 0.001x^3+0.12x^2-4x+80$$

   or

$$0 = 0.24x^2-7.072x-120$$

   so

$$x = \frac{7.072\pm\sqrt{7.072-4(0.24)(-120)}}{0.48} \approx 41.51 \text{ (the other root is negative)}$$

   Then,

$$P(41.51)-P(0) = \int_0^{41.51}(-0.24x^2+7.072x+120)\,dx = -0.08x^3+3.536x^2+120x\Big|_0^{41.51} \approx 5352.01 \text{ hundred dollars}$$

   so the maximum profit is $P(41.51) = 5352.01+P(0) = 5352.01-FC = 5352.01-300 = 5052.01$ hundred dollars or $\$505,201$.

3. (a) At $x = 3$, the value of f'' is NEGATIVE. (f' is decreasing)
   (b) On the interval $(0, 1)$ the graph of $f(x)$ is CONCAVE UP. (f' is increasing so $f'' > 0$)
   (c) The value $f(0)$ is CAN'T TELL.
   (d) On the interval $(7, 11)$ $f(x)$ is DECREASING. ($f' < 0$)
   (e) The integral $\int_{10}^{16} f'(x)\,dx$ is (POSITIVE. (look at areas)
   (f) The integral $\int_{14}^{16} f''(x)\,dx$ is POSITIVE. (this is $f'(16) - f'(14)$)
   (g) The difference $f(9) - f(2)$ is POSITIVE. (this is $\int_2^9 f'(x)dx$)
   (h) At $x = 12$, the function $f(x)$ has a MINIMUM. (f' switches sign from - to +)

4. (a)

$$f'(x) = -xe^{-0.5x^2} = 0$$

when $x = 0$. $f''(x) = -e^{-0.5x^2} + x^2e^{-0.5x^2} = (x^2 - 1)e^{-0.5x^2}$ so $f''(0) = -1 < 0$. The function is concave down at its critical point so it is a relative (and an absolute) maximum.

   (b)

$$f(x) = \int 5e^{3x} - \frac{1}{1+4x}\,dx = \frac{5}{3}e^{3x} - \frac{\ln(1+4x)}{4} + C$$

$$2 = f(0) = \frac{5}{3} + C$$

so $C = 1/3$ and $f(x) = \frac{5}{3}e^{3x} - \frac{\ln(1+4x)}{4} + \frac{1}{3}$.