Math 112
Midterm I
February 4, 2010

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<td><strong>Total</strong>:</td>
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- You are allowed to use a calculator, a ruler, and one sheet of notes.
- Besides this cover page, your exam should contain 5 problems on 4 pages. Make sure you have a complete test.
- Unless otherwise instructed, you must show how you get your answers. Answers missing supporting work may result in little or no credit, even if they are correct.
- If an algebraic method is available, answers obtained by guessing, reading values off a graphing calculator, or plug-and-check will get little or no credit.
- If you need more room, use the backs of pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

GOOD LUCK!
1 (7 points) The function $g(x)$ is given by: $g(x) = 2x^2 - x$.
   
   a) (5 pts) Write a formula in terms of $m$ and $h$ for the slope of the secant line through the graph of $g(x)$ at the points $x = m$ and $x = m + h$. Show work and simplify your answer as far as possible.

   Answer: ____________________________

   b) (2 pts) Use the answer you obtained in part (a) to compute $g'(m)$. Clearly indicate your steps.

   Answer: $g'(m) =$ __________

2 (8 points) Show work and simplify your answers.

   a) Find the derivative $f'(x)$ of the function $f(x) = (1 - x^2)(3 + 5x)$.

   Answer: $f'(x) =$ __________________________

   b) Find $\frac{ds}{dt}$, if $s = 2\sqrt{t^2} - \frac{5}{\sqrt{t}} + 3$.

   Answer: $\frac{ds}{dt} =$ __________________________
3 (10 points) The following is the graph of a function $D(t)$, representing the distance along a straight road between a remote-controlled car and its owner.

a) (3 pts) Estimate carefully the instantaneous speed of the car at 1 minute. Show your work.

Answer: __________ ft/min

b) (3 pts) Estimate the value of $\frac{D(2.6) - D(2.5)}{0.1}$. Show your work.

Answer: _______________

c) (4 pts) Sketch a rough graph of the speed of this car. Label the x-intercepts. No need to label any y-coordinates.
In this problem you don’t need to justify any of your answers.

Two balloons, A & B, start off at the same altitude of 40 yards above ground at time $t = 0$, then they move up and down. The graphs below are the corresponding rate-of-ascent graphs for the two balloons.

a) For each of the following statements circle the correct answer: True (T), False (F), or cannot tell based on the given information (CT).
   i) At $t = 2$ hours, balloon A’s altitude is higher than 40 yards. T F CT
   ii) At time $t = 7$ hrs, balloon B is higher than balloon A. T F CT
   iii) The distance between the two balloons is greater at $t = 2$ than at $t = 4$ T F CT

b) Find a 2 hour time interval, if any exists, during which balloon A is ascending but balloon B is descending.

   Answer: From $t = _____$ to $t = _______$ hours, --OR-- circle “none exists”.

c) When is the distance between the two balloons the greatest?

   Answer: At $t = ______$ hours.

d) Sketch $A(t)$, the altitude graph for balloon A. Label the y-intercept (no other y-coordinates are required)
(13 Points) The Total Revenue and Total Cost, in **hundreds of dollars**, for producing \(q\) **hundred Items** are given by the following formulas:

\[
TR(q) = -q^2 + 20q \\
TC(q) = \frac{1}{15} q^3 - \frac{25}{10} q^2 + 25q + 16
\]

a) (5 pts) Use the derivative rules to write formulas in terms of \(q\) for the Marginal Revenue and the Marginal Cost at \(q\) hundred Items.

\[
MR(q) = \text{______________________________} \\
MC(q) = \text{______________________________} \quad \text{Units for both } MR \text{ and } MC: \text{______________________________}
\]

b) (5 pts) Compute the quantity that maximizes the profit. Show work and round your answer to the nearest two decimal digits.

Answer: Profit is maximal at ______________ hundred Items

c) (3pts) Compute the maximum possible profit. Round your answer to the nearest two decimal digits.

Answer: The maximum profit is ______________ hundred dollars