# Math 112, Spring 2019, Midterm I 

April 23, 2019

Name
TA/Section

## Instructions.

- There are 4 questions. The exam is out of 40 points.
- You are allowed to use one page of notes written only on one side of the sheet in your own handwriting. It has to be the original and not a photocopy. Hand in your notes with your exam paper.
- You may only use a TI 30X IIS calculator.
- Show your work. If I cannot read or follow your work, I cannot grade it. You may not get full credit for a right answer if your answer is not justified by your work. Please BOX your final answer.

Copying from someone elses paper, using notes (unless expressly allowed by the teacher), altering an exam for re-grading, getting an advance copy of the examination, or hiring a surrogate test-taker are all flagrant violations of University policy.
Source: Student Academic Responsibility, University of Washington

| Question | points |
| :---: | :---: |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| Total |  |

1. (a) (4 points) For each quantity below, write the label (A)-(D) that corresponds to it from the picture on the right. If there is no label which corresponds to the given quantity, write "none".
(i) $f^{\prime}(s)=-\ldots------$
(ii) $\frac{f(s+r)-f(s)}{r}=$ $\qquad$
(iii) $\frac{f(s)-f(r)}{s-r}=$ $\qquad$
(iv) $f(s+r)-f(s)=$ $\qquad$

(b) Given that $f(s+r)-f(s)=\frac{2 s r+r}{(s+3)(s+r)}$
(i) (2 points) Find the average rate of change of $f(x)$ from $x=3$ to $x=5$.
(ii) (3 points) Compute $f^{\prime}(7)$.
2. Differentiate the following functions.
(a) (3 points) $f(x)=\frac{x^{3}}{7}-2 \sqrt{x}+\frac{4}{x}$. Your answers should not have negative exponents.
(b) (3 points) $f(x)=\left(7 x^{2}-5\right) \sqrt{3 x+1}$. You do not have to simplify your answer.
(c) (5 points) $f(x)=\frac{(x-2)^{2}}{\left(x^{2}+1\right)^{3}}$. Simplify your answer and find the values of $x$ where the graph of $y=f(x)$ has a horizontal tangent line.
3. At $t=0$, two balloons $A$ and $B$ are 40 feet from ground level. Their altitude functions are $A(t)$ and $B(t)$, respectively. The following are the graphs of their rates of ascent $A^{\prime}(t)$ and $B^{\prime}(t)$.

(a) (1 point) Estimate the average rate of change of altitude for Balloon B during the 0.1 second interval starting at $t=10$.
(b) (1 point) When is the distance between them maximum?
(c) (1 point) When will Balloon $B$ reach its highest altitude?
(d) (3 points) Find the instantaneous rate of change of altitude of Balloon $A$ at $t=4.34$ seconds. Be as precise as you can. Is it going up or down?
(e) (1 point) Find the interval where both balloons are going down towards the ground.
(f) (2 points) Describe the motion of Balloon $A$ as going up/going down, speeding up/slowing down in the interval $[0,7]$.
4. The Cost function for manufacturing and selling Tops is given by

$$
C(x)=0.6 x^{2}+1
$$

in hundreds of dollars where $0 \leq x \leq 4$ is in hundreds of Tops. The demand for Tops is given by the price function

$$
p=x^{2}-5.75 x+9.5
$$

where $p$ is the price in dollars per Top and the quantity $x$ is given in hundreds of Tops. Give units with your answers below.
(a) (2 points) Approximate the cost of producing the 501st Top.
(b) (3 points) At what quantity is Marginal Revenue minimum?
(c) (3 points) Do you make a profit from the sale of the 201st Top?
(d) (3 points) Find the maximum profit.

