Solutions to Math 112 Spring 2017, Midterm I

1. (a) $\dfrac{f(7)-f(4)}{7-4} = \dfrac{f(4+3)-f(4)}{3} = \dfrac{4\cdot 3+3^2-3\cdot 3}{3} = 4.$

   (b) $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \dfrac{xh+h^2-3h}{h} = \lim\limits_{h\to 0} x+h-3 = x-3$. So, $f'(4) = 4-3 = 1$.

   (c) $\dfrac{f(3)-f(0)}{3-0} = \dfrac{f(0+3)-f(0)}{3} = \dfrac{0\cdot 3+3^2+3*3}{3} = 0$. so $f(3)-(-2)=0$ and $f(3)=-2$.

2. (a) $f'(x) = -\dfrac{14}{x^3} + \dfrac{2}{\sqrt{x}}$

   (b) $f'(x) = 7\left(\dfrac{4x-7}{x^3+1}\right)^6 \dfrac{4(x^3+1)-(4x-7)3x^2}{(x^3+1)^2} = \dfrac{7(4x-7)^6(-9x^3+21x^2+4)}{(x^3+1)^8}$

   (c) $f'(x) = 2\,(9x-8)\cdot 9\cdot \left(x^2+x+1\right)^5 + (9x-8)^2\cdot 5\cdot \left(x^2+x+1\right)^4 (2x+1)$
   or $f'(x) = (9x-8)\left(x^2+x+1\right)^4 \left(108x^2-17x-22\right)$

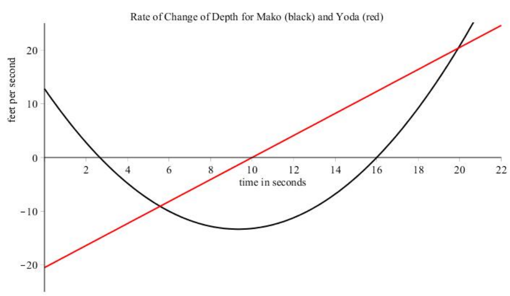
3. (a) $s'(t) = 0.3t^2-5.6t+12.8$ so $s'(12) = -11.2$ feet per second. He is swimming up (depth is decreasing) at 11.2 feet per second.

   (b) When s' switches from + to -: $s'(t)=0$ when

   $$t = \frac{5.6 \pm \sqrt{5.6^2-4(0.3)(12.8)}}{0.6} = 16 \text{ or } 8/3 \approx 2.67.$$

   The graph of s' is a parabola which opens up (see below) so the switch from + to - is at $t=8/3$ seconds.

   (c) The graph of s' is a parabola which opens up. It crosses the time axis at $t=8/3$ and $t=16$. You can see the y intercept to be 12.8 from the equation. Yoda is swimming fastest at the vertex of the parbola when $t = 6.6/0.6 \approx 9.3$ seconds. (You can also find this by $s''=0$). To complete the picture of the parabola you can compute his velocity $s'(28/3) \approx -13.3$ seconds. Since this is negative, his depth is decreasing and he is swimming towards the surface.



   (d) See above picture.

   (e) They start at the same depth at $t=0$ and then until $t\approx 2.67$, Mako dives and Yoda swims up so they are moving further apart at $t=2$.

   (f) Yoda, by the above argument.

4. (a) $C'(x) = 0.75x^2-12x+60$ so $C'(5) = 18.75$ dollars per Top.

   (b) $P'(x) = R'(x)-C'(x) = (-5x+80)-(0.75x^2-12x+60) = -0.75x^2+7x+20 = 0$ when

   $$x = \frac{-7 \pm \sqrt{49+4(0.75)20}}{-1.5} \approx 11.627 \text{ or } -2.29.$$

   P' is a parabola which opens down so the switch from + to - happens at $x \approx 11.627$. So the maximum profit is $P(11.627) \approx 192.740$ thousand dollars or $\$192,740$.

   (c) $MC' = 1.5x-12 = 0$ when $x = 8$ thousand Tops.