Solutions to Spring 2019 Math 112 Final

1. (a) $\dfrac{d}{dx}\left(\dfrac{e^{7x}}{(2x+5)^2}\right) = \dfrac{7e^{7x}(2x+5)^2 - 4e^{7x}(2x+5)}{(2x+5)^4}$

   (b) $\dfrac{d}{dx}\left(\ln(2x^3+5x-2)\right) = \dfrac{6x^2+5}{2x^3+5x-2}$

   (c) $\dfrac{d}{dx}\left(e^{5x}(2x^3-1)^9\right) = 5e^{5x}(2x^3-1)^9 + 54x^2e^{5x}(2x^3-1)^8$

   (d) $\dfrac{\partial}{\partial x}\left((3x^2+6y^3)e^{x+y}\right) = 6xe^{x+y} + (3x^2+6y^3)e^{x+y}$

   (e) $\dfrac{\partial}{\partial y}\left(\dfrac{7x^3y^2-6xy}{7x-8y}\right) = \dfrac{(14x^3y-6x)(7x-8y)+8(7x^3y^2-6xy)}{(7x-8y)^2}$

2. (a) $\displaystyle\int \frac{x^2-1+\sqrt{x}}{x}\,dx = \frac{x^2}{2} - \ln|x| + 2\sqrt{x} + C$

   (b) $\displaystyle\int e^{2x}\left(1-e^{5x}\right)\,dx = \frac{e^{2x}}{2} - \frac{e^{7x}}{7} + C$

   (c) $\displaystyle\int_1^4 3\sqrt{x} - 5x + \frac{1}{x}\,dx = 2x^{3/2} - \frac{5}{2}x^2 + \ln|x|\Big|_1^4 = \ln 4 - \frac{47}{2}$

3. $f(x) = 2x^5 + 5x^4 - 10x^3 + 7.$

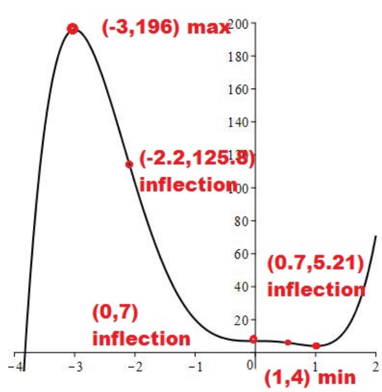
   (a)
   $$f'(x) = 10x^4 + 20x^3 - 30x^2 = 10x^2(x+3)(x-1)$$
   So the critical numbers are $x, 0, 1, -3$. Checking the sign of f' in the intervals $(-\infty,-3)$ (positive), $(-3,0)$ (negative), $(0,1)$ (negative) and $(1,\infty)$ (positive), we conclude that $x=-3$ gives a local max, $x=0$ gives a horizontal inflection and $x=1$ gives a local min.

   (b) From above on $(infty,-3)$ and $(1,\infty)$.

   (c) Find the x-coordinates of the inflection points of $f(x)$.
   $$f''(x) = 40x^3 + 60x^2 - 60x = 20x(2x^2+3x-3)$$
   Using the quadratic formula
   $$x = \frac{-3\pm\sqrt{33}}{4}$$
   and $x=0$ are the x-coordinates of the inflection points.

   (d) Checking the sign of the second derivative on the intervals $\left(-\infty, \frac{-3-\sqrt{33}}{4}\right)$ (negative), $\left(\frac{-3-\sqrt{33}}{4}, 0\right)$ (positive), $\left(0, \frac{-3+\sqrt{33}}{4}\right)$ (negative), $\left(\frac{-3+\sqrt{33}}{4}, \infty\right)$ (positive), we conclude that the graph of f is concave up on the intervals $\left(\frac{-3-\sqrt{33}}{4}, 0\right)$ and $\left(\frac{-3+\sqrt{33}}{4}, \infty\right)$.

   (e) One of the graphs below belongs to $f(x)$. Find it and mark points you found in parts (a) and (c) in the form (x,y) on it.



4.
$$MC(x) = 0.0125x^2 - 0.15x + 3.15 \quad \text{and} \quad TR(x) = -0.0042x^3 + 0.15x^2 + 5.6x$$

(a)
$$MR(x) = -0.0126x^2 + 0.3x + 5.6$$

has its maximum when
$$MR'(x) = -0.0252x + 0.3 = 0$$

so when $x = 11.90$ units with $MR(11.90) = 7.39$ dollars per bottle.

(b) What is the Variable Cost of producing 17 hundred bottles?

$$VC(17) = VC(17) - VC(0) = \int_0^{17} 0.0125x^2 - 0.15x + 3.15\, dx = \frac{0.0125}{x}x^3 - \frac{0.15}{2}x^2 + 3.15x\Big|_1^{17} = 52.3458 \text{ hundred dollar}$$

or \$5234.58.

(c) When
$$0.0125x^2 - 0.15x + 3.15 = -0.0126x^2 + 0.3x + 5.6$$

cleaning up and using the quadratic formula $x \approx 22.30$.

(d)

$$P(22.30) - P(0) = \int_0^{22.30} (-0.0126x^2 + 0.3x + 5.6) - (0.0125x^2 - 0.15x + 3.15)\, dx = 73.74 \text{ hundred dollars}$$

So, maximum profit is

$$P(22.30) = 73.74 + P(0) = 73.74 - 12.5 = 61.24 \text{ hundred dollars.}$$

5. (a) $a(5) = 0$, it has momentarily stopped.

(b)
$$\int_0^3 a(t)\, dt = 3 \times 5 + \frac{2 \times 5}{2} = 20 \text{ meters}$$

(c)
$$A(5) - A(3) = \int_3^5 a(t)\, dt = 7.5 \text{ meters}$$

Average rate of change is

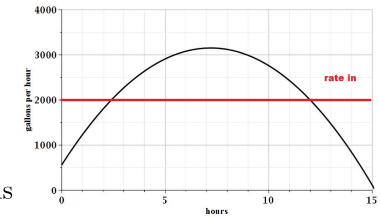
$$\frac{A(5) - A(3)}{5 - 3} = 3.75 \text{ meters per minute}$$

(d) When $A'' = a' > 0$ so when $a(t)$ is increasing : $(8, 12)$

(e) At $t = 5$.

(f)
$$1.6 + \int_0^7 a(t)\, dt = 1.6 + 20 + 7.5 - 7.5 = 21.6 \text{ meters}$$

6.



(a) See picture

(b)

$$\int_0^6 -50t^2 + 720t + 560\,dt = 12720 \text{ gallons}$$

(c) When $r'(t) = -100t + 720 = 0$ so $t = 7.2$ hours. The maximum rate is $r(7.2) = 3152$ gallons per hour.

(d) Critical numbers are when

$$2000 = -50t^2 + 720t + 560$$

so $t = 12$ or $t = 2.4$. Now we need to check the amount $A(t)$ at these critical numbers and at $t = 0$ and $t = 15$. First, we calculate the amount $A(t)$:

$$A(t) = \int 2000 - (-50t^2 + 720t + 560)\,dt = \frac{50}{3}t^3 - 360t^2 + 1440t + C$$

Since $A(0) = 12,000$, $C = 12,000$. Now compare values:

$$A(0) = 12,000, A(15) = 8850, A(2.4) = 13612.8, A(12) = 6240$$

So the minimum amount is 6240 gallons and the maximum amount is 13,612.8 gallons.

7. (a)

$$p(24) = 25e^6$$

$$PS = 24(25e^6) - \int_0^{24} 25e^{x/4}dx = 600e^6 - \left(100e^{x/4}\Big|_0^{24}\right) = 500e^6 + 100 \approx 201,814.40 \text{ dollars.}$$

(b) Equilibrium point:

$$25 = \frac{450}{2x+8} \rightarrow \qquad x = 5.$$

$$\text{CS} = \int_{)}^{5} \frac{450}{2x+8}\,dx\ - 5 \cdot 25 = 225\ln(2x+8)\Big|_0^5 - 125 = 225\,(\ln 18 - \ln 8) - 125 \approx 57.50 \text{ dollars}$$

8.

$$P(x,y) = 6x + 9y - (2x^2 - 2xy + y^2 - 8x - 9y + 11) = -2x^2 + 2xy - y^2 + 14x + 18y - 11$$

Critical point:

$$P_x = -4x + 2y + 14 = 0$$

$$P_y = 2x - 2y + 18 = 0$$

Solving the system we get $(x, y) = (16, 25)$ so the maximum profit is $P(16, 25) = 326$ dollars.

Find the maximum profit assuming that profit will be maximized at its critical point, .