MATH 112
FINAL EXAM
JUNE 8, 2019

Name $\qquad$
Student ID \# $\qquad$ Quiz Section $\qquad$

## HONOR STATEMENT

"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam."

## SIGNATURE:

| 1 | 15 |  |
| :---: | :---: | :--- |
| 2 | 12 |  |
| 3 | 14 |  |
| 4 | 15 |  |
| 5 | 10 |  |
| 6 | 15 |  |
| 7 | 10 |  |
| 8 | 9 |  |
| Total | 100 |  |

- Your exam should consist of this cover sheet, followed by 8 problems on 8 pages. Check that you have a complete exam.
- Turn your cell phone off and put it away for the duration of the exam. You may not listen to headphones or ear buds during the exam.
- You may use a Ti-30x IIS calculator. No other electronic items are to be used during the exam.
- You may have one handwritten $8.5 \times 11$ sheet of notes; writing is allowed on both sides.
- You must show all of your work. The correct answer with little or no supporting work may result in no credit.
- Unless otherwise indicated, you may round your final answer to two digits after the decimal.
- You've signed an honor statement. Don't cheat.

1. (15 points) Compute the following. Make sure you have used parentheses correctly.
(a) $\frac{d}{d x}\left(\frac{e^{7 x}}{(2 x+5)^{2}}\right)$
(b) $\frac{d}{d x}\left(\ln \left(2 x^{3}+5 x-2\right)\right)$
(c) $\frac{d}{d x}\left(e^{5 x}\left(2 x^{3}-1\right)^{9}\right)$
(d) $\frac{\partial}{\partial x}\left(\left(3 x^{2}+6 y^{3}\right) e^{x+y}\right)$
(e) $\frac{\partial}{\partial y}\left(\frac{7 x^{3} y^{2}-6 x y}{7 x-8 y}\right)$
2. (12 points) Compute the following.
(a) $\int \frac{x^{2}-1+\sqrt{x}}{x} d x$
(b) $\int e^{2 x}\left(1-e^{5 x}\right) d x$
(c) $\int_{1}^{4} 3 \sqrt{x}-5 x+\frac{1}{x} d x$
3. (14 points) Let $f(x)=2 x^{5}+5 x^{4}-10 x^{3}+7$.
(a) Find all critical numbers for $f(x)$ and determine if they give a local minimum, local maximum or a horizontal inflection.
(b) Find the interval(s) where the graph of $f(x)$ is increasing.
(c) Find the $x$-coordinates of the inflection points of $f(x)$.
(d) Find the interval(s) where the graph of $f(x)$ is concave up.
(e) One of the graphs below belongs to $f(x)$. Find it and mark points you found in parts (a) and (c) in the form $(x, y)$ on it.



4. ( 15 points) You produce and sell Love Potions. (Number 9 is a best seller.) The Marginal Cost and Total Revenue functions are given by

$$
M C(x)=0.0125 x^{2}-0.15 x+3.15 \quad \text { and } \quad T R(x)=-0.0042 x^{3}+0.15 x^{2}+5.6 x
$$

where $x$ is in hundred of bottles, $M C$ is in dollars per bottle and $T R$ is in hundreds of dollars. Round your answers to the nearest bottle and nearest dollar. Include units.
(a) What is the maximum value of Marginal Revenue?
(b) What is the Variable Cost of producing 17 hundred bottles?
(c) At what quantity is the profit maximized?
(d) If fixed costs are $\$ 1,250$, what is the maximum profit?
5. (10 points) Below is the graph of $a(t)$, the rate of change of altitude of an umbrella flying in the wind, where time $t$ is in minutes and the rate of change of height is in meters per minute.

(a) How fast is the umbrella moving at $t=5$ ? Is it rising or falling?
(b) How much does the umbrella rise during the first 3 minutes?
(c) What is the average rate of change of its altitude between $t=3$ and $t=5$ minutes?
(d) Let $A(t)$ be the altitude function of the umbrella. In which interval(s) is the graph of $A(t)$ concave up?
(e) When does the umbrella reach its maximum height during the 12 minute interval shown on the graph?
(f) If initially the umbrella is 1.6 meters above the ground, what is its altitude at $t=7$ ?
6. A water recycling system has a very large tank (it won't overflow over during this problem) where water to be recycled is pumped in and pure water is pumped out. Dirty water is pumped into the tank at a constant rate of 2000 gallons per hour. Pure water is pumped out of the tank at a rate of

$$
r(t)=-50 t^{2}+720 t+560
$$

gallons per hour. Initially there are 12,000 gallons of water in the tank. On the right is the graph of $r(t)$. Both pumps stop at $t=15$.

(a) (1 point) Graph the rate at which water gets pumped into the tank on the graph above to help you with the questions below. You can compare approximate values from the graph with your computations below.
(b) (4 points) How much water gets pumped out of the tank during the first 6 hours?
(c) (3 points) When is the water being pumped out of the tank at the fastest rate? How fast is water being pumped out at this time?
(d) (7 points) What are the minimum and maximum amounts of water in the tank?
7. (10 points) The two parts of this question are computationally unrelated. Round your answers to the nearest cent.
(a) The supply function for a product is given by

$$
p=25 e^{x / 4}
$$

where $x$ is the number of units and $p$ is in dollars per unit. The equilibrium quantity is 24 units. Find the producer's surplus at equilibrium.
(b) The demand function for a different product is given by

$$
p=\frac{450}{2 x+8}
$$

where $x$ is the number of units and $p$ is in dollars per unit. The equilibrium price is $\$ 25$ per unit. Find the consumer's surplus at equilibrium.
8. ( 9 points) A bakery makes meringue pies and creme brulees. If $x$ is the number meringue pies and $y$ is the number of creme bruless, then the total cost and total revenue in dollars are

$$
C(x, y)=2 x^{2}-2 x y+y^{2}-8 x-9 y+11
$$

and

$$
R(x, y)=6 x+9 y
$$

Find the maximum profit assuming that profit will be maximized at its critical point, .

