Math 112
Final Exam
June 3, 2017

Name $\qquad$
Student ID \# $\qquad$ Section $\qquad$

## HONOR STATEMENT

"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam."

## SIGNATURE:

| 1 | 12 |  |
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| 2 | 11 |  |
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| 5 | 9 |  |
| 6 | 12 |  |
| 7 | 15 |  |
| 8 | 12 |  |
| Total | 100 |  |

- Your exam should consist of this cover sheet, followed by 8 problems on 8 pages. Check that you have a complete exam.
- Turn your cell phone OFF and put it away for the duration of the exam.
- You may not listen to headphones or ear buds during the exam.
- Unless otherwise indicated, you must use the methods of this course and show all of your work. The correct answer with little or no supporting work may result in no credit.
- You can use a hand written sheet of notes and a TI 30X IIS calculator.
- Follow the instructions on individual questions on how to round your answers.
- You have signed an honor statement. Do not cheat.

1. Compute the following.
(a) (4 points) $\frac{d}{d x} \ln \left(e^{x} \sqrt{3 x+1}\right)=$
(b) $(4$ points $) \int\left(x^{3}-5\right)\left(7 x-\frac{2}{x}\right) d x=$
(c) (4 points) $\int_{0}^{1} \frac{e^{2 x}-7}{e^{x}} d x=$
2. The marginal revenue and marginal cost at $x$ hundred things are given by the graphs below.

(a) (3 points) Does your profit increase or decrease when you sell the 2001st Thing? By approximately how much?
(b) (3 points) Approximate the fixed cost if it costs $\$ 6650$ to produce 1300 Things.
(c) (3 points) Estimate the minimal profit (maximal loss) and the quantity where it occurs.
(d) (2 points) Approximately when do you break even? Support your answer with calculations.
3. Two dogs Copper and Penny are out running. The distance of Copper to the house is given by

$$
C(t)=0.125 t^{2}+0.75 t
$$

meters with $t$ in seconds. The rate of change of distance (the velocity) of Penny is given by

$$
p(t)=0.15 t^{2}-2.4 t+7.2
$$

meters per second. Round your answers to three digits after the decimal for this question.
(a) (5 points) When is Penny running the fastest during the first 20 seconds? Is she running towards the house or away from the house?
(b) (5 points) At $t=9$, they (almost) run into each other. Where is Penny at $t=0$ ?
(c) (4 points) What is the maximum distance between them during the first 10 seconds?
4. This question analyzes the behavior of the function $f(x)=x^{3}-3 x^{2}-9 x+15$ and ends up with a sketch of its graph.
(a) (4 points) Find the critical points of $f(x)$ and identify them as a local minimum, a local maximum or a horizontal point of inflection.
(b) (2 points) In which interval(s) is the function increasing?
(c) (3 points) In which interval(s) is the graph of $f(x)$ concave up?
(d) (4 points) Find the coordinates $(x, y)$ of all relative maximum, relative minimum and inflection points and mark them on the axes below. Also compute and mark the $y$ intercept.
(e) (2 points) Using the points you marked and the information you collected in parts (a)-(c) make a sketch of the graph of $f(x)$ on the right. I will be looking to see if the increasing/decreasing and concave up/concave down information you gave above matches with your picture on the right.

5. A large corporation with monopolistic control in the marketplace has its average daily costs, in dollars per unit, given by

$$
A C(q)=\frac{800}{q}+200 q+10 q^{2}
$$

The daily demand for $q$ units of its product is given by

$$
p=24000-100 q
$$

dollars per unit.
(a) (7 points) Find the quantity that gives maximum profit and find the maximum profit.
(b) (2 points) What selling price should the corporation set for its product to maximize its profit?
6. The demand for oyster flavored dog biscuits is given by the function

$$
p=\frac{63}{0.05 x+2}
$$

where $x$ is the quantity in cases of biscuits and $p$ is the price in dollars of a case of biscuits. The supply function is

$$
p=0.9+1.35 x .
$$

(a) (8 points) Find the consumer's surplus. Round your answer to three decimal places.
(b) (4 points) The following are graphs of the supply and demand functions given above. Mark them with $\mathbf{S}$ and $\mathbf{D}$. Show the area which gives the consumer's surplus you computed above. Estimate the area from the graph and and verify that it is close to your answer above. They will not be the same.

7. Consider the function

$$
f(x, y)=3 x^{2}+\ln (y+1)+e^{0.2 x y}
$$

(a) (3 points) Compute $f_{x}(x, y)$.
(b) (3 points) Compute $f_{y}(x, y)$.
(c) (2 points) Circle the greater one:

$$
f_{x}(2,3) \quad \text { OR } \quad f_{y}(2,3)
$$

(d) (2 points) Circle the greater one:

The slope of $f(1, y)$ at $y=2 \quad$ OR $\quad$ The slope of $f(x, 2)$ at $x=1$
(e) (2 points) Circle the greater one:

$$
\frac{f(7.000000001,3)-f(7,3)}{0.000000001} \quad \text { OR } \quad \frac{f(3,7.000000001)-f(3,7)}{0.000000001}
$$

(f) (3 points) If $x$ is thousands boxes of mint ice cream and $y$ is the thousands of boxes of chocolate ice cream produced by Delighted, and $f(x, y)$ is their cost in thousands of dollars, compute $f_{x}(2.7,4.3)$ and say what it means without using any calculus terminology.
8. (12 points) We Love Music manufactures two products, xylophones and yang qins. If $x$ is the number of thousands of units of xylophones and $y$ is the number of thousands of units of yang qins, then the total cost and total revenue in thousands of dollars are

$$
C(x, y)=2 x^{2}-2 x y+y^{2}-7 x-9 y+13, \quad R(x, y)=9 x+5 y .
$$

Find the maximum profit assuming the profit function will be maximized at its critical point.

