1. You produce and sell Things. For the production, the Marginal Cost and the Average Variable Cost are given in terms of q things by

$$MC = 0.021q^2 - 2.1q + 75 \qquad\qquad AVC = 0.007q^2 - 1.05q + 75$$

dollars per Thing. The Fixed Cost is $500. For selling Things, the Price p and the Marginal Revenue are given in terms of the quantity q Things by

$$p = -0.008q^2 + 0.96q + 21.6 \qquad\qquad MR = -0.024q^2 + 1.92q + 21.6$$

dollars per Thing.

(a) (2 points) Give the formulas for the following:

price × quantity = The Total Revenue $TR = -0.008q^3 + 0.96q^2 + 21.6q$

$q \times AVC(q) + FC$ = The Total Cost $TC = 0.007q^3 - 1.05q^2 + 75q + 500$

(b) (4 points) What is the Shutdown Price?

lowest value of AVC: OR

$$q = \frac{-(-1.05)}{2(0.007)} = 75$$

$$SP = AVC(75) = 35.625$$

$MC = AVC$

$$0.021q^2 - 2.1q + 75 = 0.007q^2 - 1.05q + 75$$
$$0.028q^2 - 1.05q = 0$$
$$(0.028q - 1.05)q = 0$$
$$q \not= 0 \quad OR \quad q = \frac{1.05}{0.028} = 75$$

$$SP = AVC(75) = MC(75) = 35.625$$

(c) (5 points) What is the Maximum Profit?

$(P(q) = TR(q) - TC(q)$ is **cubic** so I can't find its max$)$

P is max when $MC = MR$

$$0.021q^2 - 2.1q + 75 = -0.024q^2 + 1.92q + 21.6$$
$$0.045q^2 - 4.02q + 53.4 = 0$$

$$q = \frac{4.02 \pm \sqrt{4.02^2 - 4(0.045)(53.4)}}{0.09} \approx 73.1 \quad OR \quad 16.2$$

optional explanation: (If you check $MC(20) = 41.4$
$\qquad\qquad MR(20) = 50.4$

you can see from $MR > MC$ to $MR > MC$ when $16.2 < q < 73.1$ so the switch $MR < MC$ happens at $q = 73.1$).

max profit = $P(73.1) = TR(73.1) - TC(73.1)$
(OR P(73)) $\qquad = 3583.88 - 3106.03 = 477.85$

2

2. On the right are graphs of $f(x) = -x^2 + 10x$ and $g(x) = x^2 - 16x + 72$. Estimating coordinates from the graph is not considered a complete solution. However, you can check your algebraic solutions by looking at the graph.
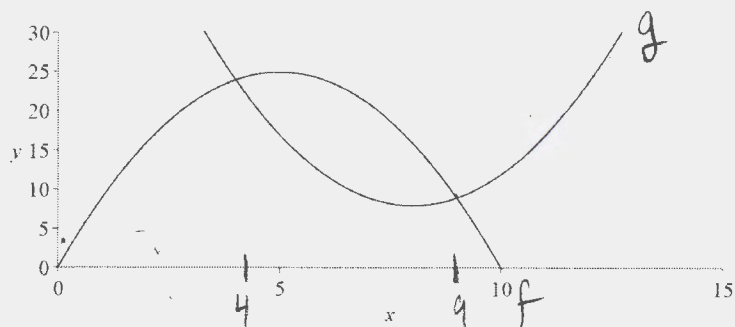
(a) (1 point) Label the functions on the graph.

(b) (5 points) Let $y = mx + b$ be the line that passes through the two intersection points of the parabolas. Find m and b.

intersection $f(x) = g(x)$

$-x^2 + 10x = x^2 - 16x + 72$

$0 = 2x^2 - 26x + 72 = 2(x^2 - 13x + 36)$

$x = \dfrac{13 \pm \sqrt{13^2 - 4(36)}}{2} = \dfrac{13 \pm 5}{2} = 9 \text{ or } 4$

$f(4) = g(4) = -16 + 40 = 24$

$f(9) = g(9) = 81 - 144 + 72 = 9$

Line Equation:

Points $(4, 24), (9, 9)$

slope $= \dfrac{24 - 9}{4 - 9} = \dfrac{15}{-5} = -3$

equation $y - 9 = -3(x - 9)$

simplified: $y = -3x + 36$

(c) (3 points) Find the interval where both $f(x)$ and $g(x)$ are decreasing.

f is decreasing after its vertex: $x = \dfrac{-10}{-2} = 5$

so $x > 5$

g is decreasing before its vertex: $x = -\dfrac{(-16)}{2} = 8$
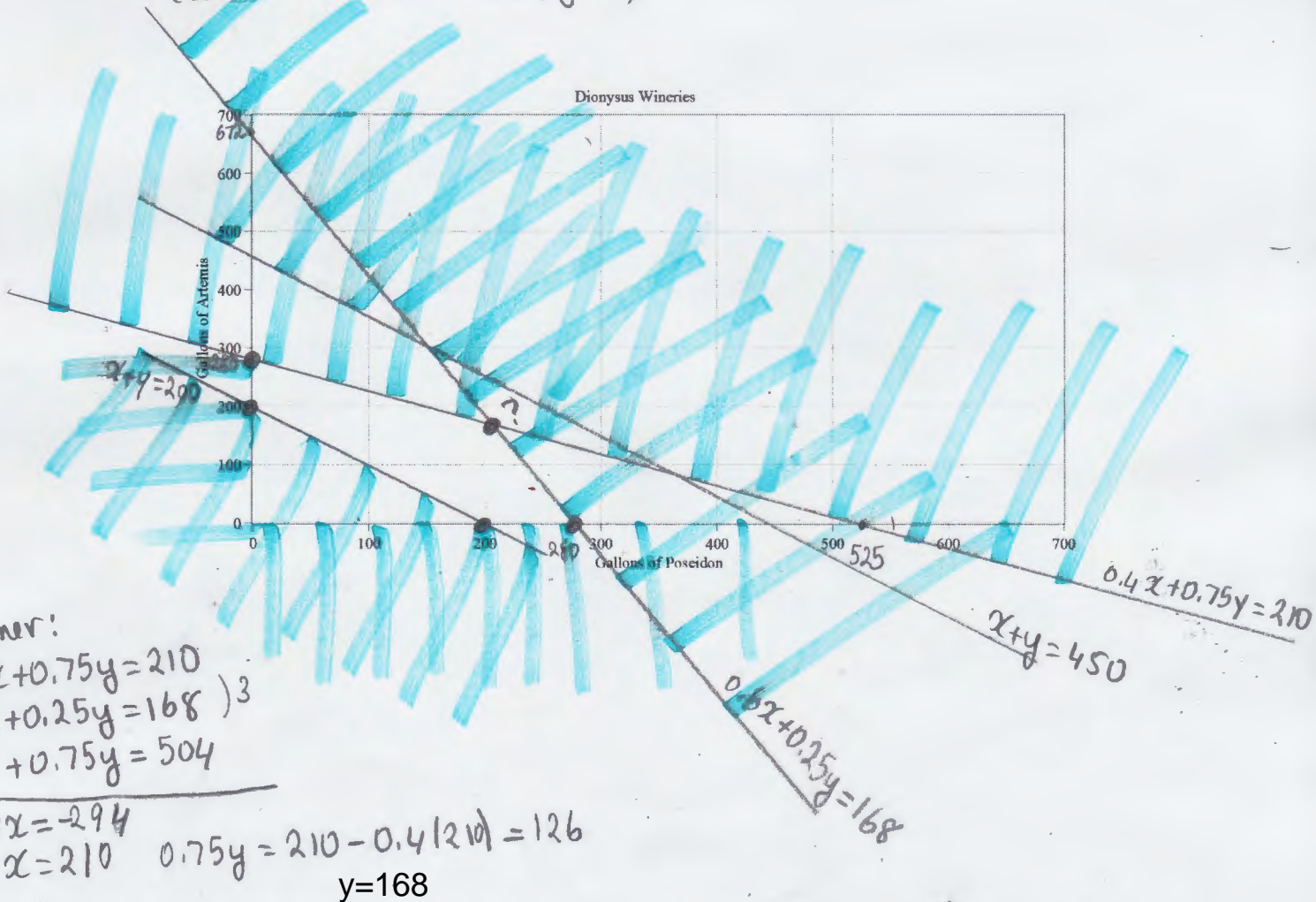
so $x < 8$

The overlap is $\boxed{5 < x < 8}$

3

3. Dionysus Wineries in Tenedos produces two types of blended wines, Poseidon and Artemis, by blending two types of grape varieties, Shiraz and Muscat. Poseidon has 40% Shiraz and 60% Muscat and Artemis has 75% Shiraz and 25% Muscat. The winery does not grow its own grapes. They can get at most 210 gallons of Shiraz juice from one vineyard and at most 168 gallons of Muscat juice from another vineyard. They have to produce at least 200 gallons of wine to fulfill their existing contracts with restaurants. Their maximum capacity for production is 450 gallons.

(a) (7 points) Let x be the gallons of Poseidon blend and y be the gallons of Artemis blend the winery produces. List all the constraints and graph the feasible region below. List all the corners of your feasible region.

Shiraz: $0.40x + 0.75y \leq 210$

Muscat: $0.60x + 0.25y \leq 168$

production $200 \leq x+y \leq 450$

(common sense: $x \geq 0, y \geq 0$)

$(0,280)$ $(525,0)$
$(0,672)$ $(280,0)$
$(0,200)$ $(200,0)$ $(0,450)$ $(450,0)$

? corner:

$0.4x + 0.75y = 210$

$(0.6x + 0.25y = 168) \cdot 3$

$1.8x + 0.75y = 504$

──────────

$-1.4x = -294$

$x = 210$ $0.75y = 210 - 0.4(210) = 126$

y=168

(b) (3 points) They make a profit of 35 dollars on every gallon of Poseidon and a profit of 27 dollars on every gallon of Artemis. What is the production level for Poseidon and Artemis blends that will maximize the profit of the winery?

corners	$(0,200)$	$(0,280)$	$(210, 168)$	$(280,0)$	$(200,0)$
$P = 35x + 27y$	5400	7560	11886	9800	7000

↑

210 gallons of Poseidon

168 gallons of Artemis

4. If you invest $15,500 in an account with 6% annual interest compounded monthly, the amount in your account after t years is given by

$$A = 15500(1.005)^{12t}.$$

(a) (5 points) How much do you have in your account after 3 years?

$$A(3) = 15500(1.005)^{12 \times 3}$$

$$= 18548.55$$

(b) (5 points) When will you double the money you have invested?

$$2(15500) = 15500(1.005)^{12t}$$

$$2 = 1.005^{12t}$$

$$\ln 2 = \ln(1.005)^{12t} = 12t \ln(1.005)$$

$$11.58 \approx \frac{\ln 2}{12 \ln(1.005)} = t$$