

Math 111 Winter 2017, Midterm II

February 28, 2017

Name Solutions

TA/Section _____

Instructions.

- There are 4 questions. The exam is out of 40 points.
- You are allowed to use one page of notes written only on one side of the sheet in your own handwriting. It has to be the original and not a photocopy. **Hand in your notes with your exam paper.**
- You may only use a TI 30X IIS calculator.
- **Show your work.** If I cannot read or follow your work, I cannot grade it. You may not get full credit for a right answer if your answer is not justified by your work.

Copying from someone else's paper, using notes (unless expressly allowed by the teacher), altering an exam for re-grading, getting an advance copy of the examination, or hiring a surrogate test-taker are all flagrant violations of University policy.

Source: Student Academic Responsibility, University of Washington

Question	points
1	
2	
3	
4	
Total	

1. (12 points) You produce and sell Louloudis. The total cost of producing q thousand Louloudis is given by

$$TC(q) = 0.25q^3 - 2.2q^2 + 8.95q + 3$$

thousand dollars. The marginal cost and marginal revenue are given by

$$MC(q) = 0.75q^2 - 4.4q + 8.95$$

$$MR(q) = -0.1q^2 + 4q + 5.25,$$

both in dollars per Louloudi.

- (a) Compute the shutdown price. Round your answer to the nearest cent.

SP = lowest value of AVC OR common value of $AVC = MC$

$$AVC = 0.25q^2 - 2.2q + 8.95$$

$$q = \frac{2.2}{2(0.25)} = 4.4$$

$$0.25q^2 - 2.2q + 8.95 = 0.75q^2 - 4.4q + 8.95$$

$$0 = 0.5q^2 - 2.2q$$

$$0 = q(0.5q - 2.2)$$

$$q = \frac{2.2}{0.5} = 4.4$$

$$SP = AVC(4.4) = 0.25(4.4)^2 - 2.2(4.4) + 8.95 (= MC(4.4))$$

$$= 4.11 \text{ dollars}$$

- (b) Find the interval where the average variable cost is decreasing and the marginal cost is increasing. Round your answers to the nearest Louloudi.

\downarrow AVC decreasing $0 \leq q \leq 4.4$
 \uparrow MC increasing $q > 2.933$
 $\frac{-(-4.4)}{2(0.75)} = 2.933$, $q > 2.933$
 so AVC \downarrow , MC \uparrow on $2.933 \leq q \leq 4.4$

- (c) At how many thousand Louloudis will you maximize profits?

When $MC = MR$

$$0.75q^2 - 4.4q + 8.95 = -0.1q^2 + 4q + 5.25$$

$$0.85q^2 - 8.4q + 3.7 = 0$$

$$q = \frac{8.4 \pm \sqrt{8.4^2 - 4(0.85)(3.7)}}{2(0.85)} = 9.420 \text{ OR } 0.462$$

which one?
0.462

$MC > MR$
(try $q=0$)
Profits \downarrow

$MC < MR$
(try $q=5$)
Profits \uparrow

$MC > MR$
(try $q=10$)
Profits \downarrow

2

Profits will maximize at
9.420 thousand Louloudis.

2. (10 points) The Corner Store will buy 200 cups from a wholesaler if the price is \$12 and 600 cups if the price is \$8. The supply function for the wholesaler is given by $100p - 3q = 75$.

(a) Find the demand function for this market of cups.

(q, p) $(200, 12)$ $(600, 8)$ slope = $\frac{12-8}{200-600} = -0.01$

$$p - 12 = -0.01(q - 200)$$

$$p = -0.01q + 14$$

(b) If the government levies a tax of \$1.25 per cup on the wholesaler, who passes the tax on to the Corner Store as a price increase, find the equilibrium price and quantity after the tax is levied.

Supply

$$100p - 3q = 75$$

$$100p = 3q + 75$$

$$p = 0.03q + 0.75$$

After tax $p = 0.03q + 2$

Equilibrium point

$$-0.01q + 14 = 0.03q + 2$$

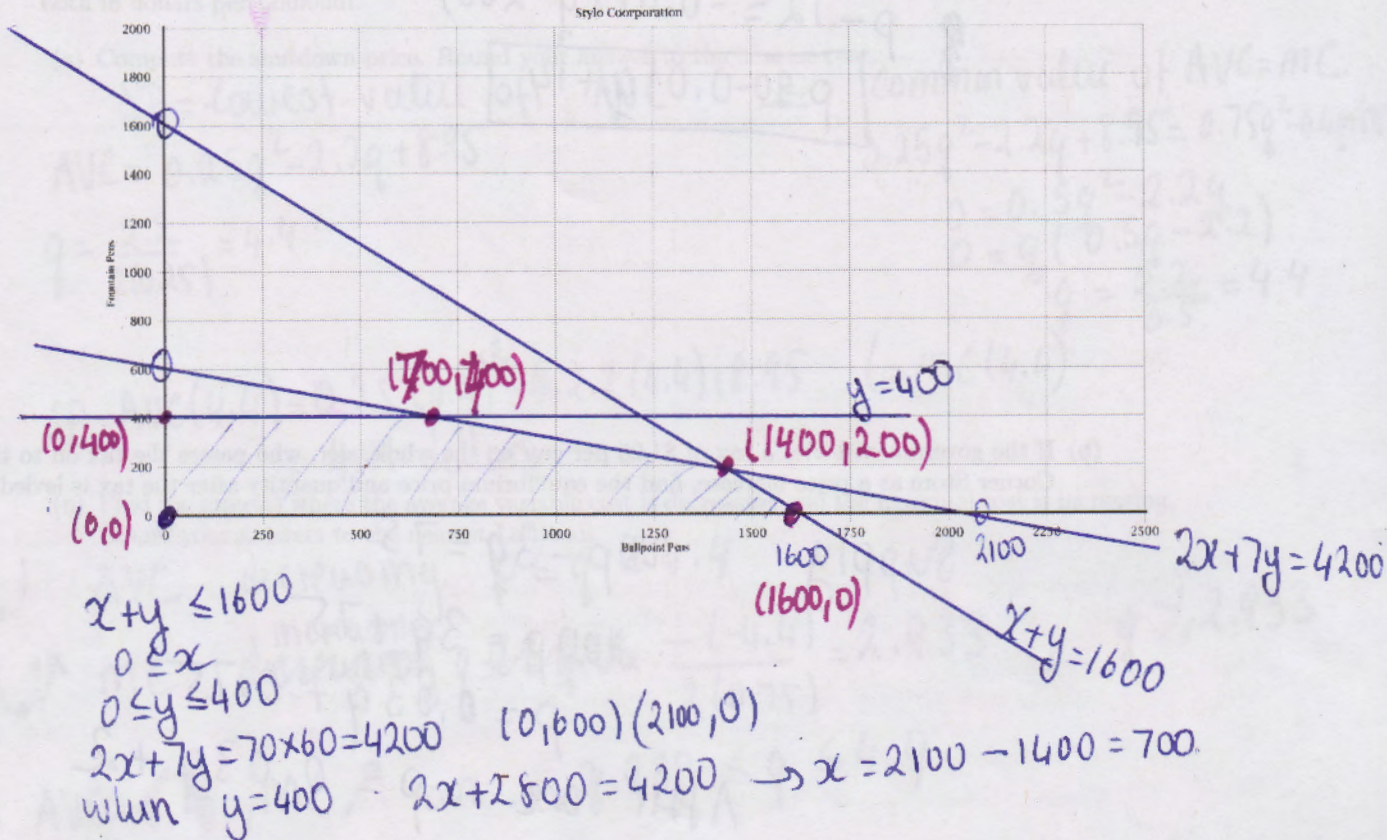
$$12 = 0.04q$$

$$300 = q$$

$$p = -0.01(300) + 14 = 11.$$

3. (10 points) Stylo Corporation manufactures ballpoint pens and fountain pens. The company has the capacity to make 1600 pens per day, with no more than 400 fountain pens per day. It has 70 hours of labor per day available. It takes 2 ~~minutes~~ minutes to make a ballpoint pen and 7 ~~minutes~~ minutes to make a fountain pen.

(a) Let x be the number of ball point pens and y be the number of fountain pens Stylo produces. Write down the inequalities and use the axes provided below to graph the feasible region for this production.



(b) The profits are 12 cents per ballpoint pen and 65 cents per fountain pen. Find the maximum profit and the number of each type of pen that should be made to give this maximum profit.

$$\begin{array}{r}
 x + y = 1600 \rightarrow 2x + 2y = 3200 \\
 2x + 7y = 4200 \quad -2x + 7y = 4200 \\
 \hline
 -5y = -100 \rightarrow y = 200 \rightarrow x = 1600 - 200 = 1400
 \end{array}$$

(x, y)	$(0,0)$	$(0,400)$	$(700,400)$	$(1400,200)$	$(1600,0)$
$P = 12x + 65y$ cents	0	260	344	298	192
OR $P = 0.12x + 0.65y$ dollars	0	260	344	298	192

max profit of \$ 344
 with 700 ballpoint, 400 fountain.

4. (8 points) If \$19,500 is invested at 4.6% compounded continuously, the future value S at any time t (in years) is given by the following formula. (Round your answers to two decimal places.)

$$S = 19500e^{0.046t}$$

- (a) What is the amount after 45 months?

$$\begin{aligned} 45 \text{ months} &= 3.75 \text{ years} \\ S(3.75) &= 19500 e^{0.046(3.75)} \\ &= \$23171.30 \end{aligned}$$

- (b) How long before the investment triples?

$$\begin{aligned} 3 \times 19500 &= 19500 e^{0.046t} \\ 3 &= e^{0.046t} \\ \ln 3 &= 0.046t \\ \frac{\ln 3}{0.046} &= t \\ t &\approx 23.88 \text{ years.} \end{aligned}$$