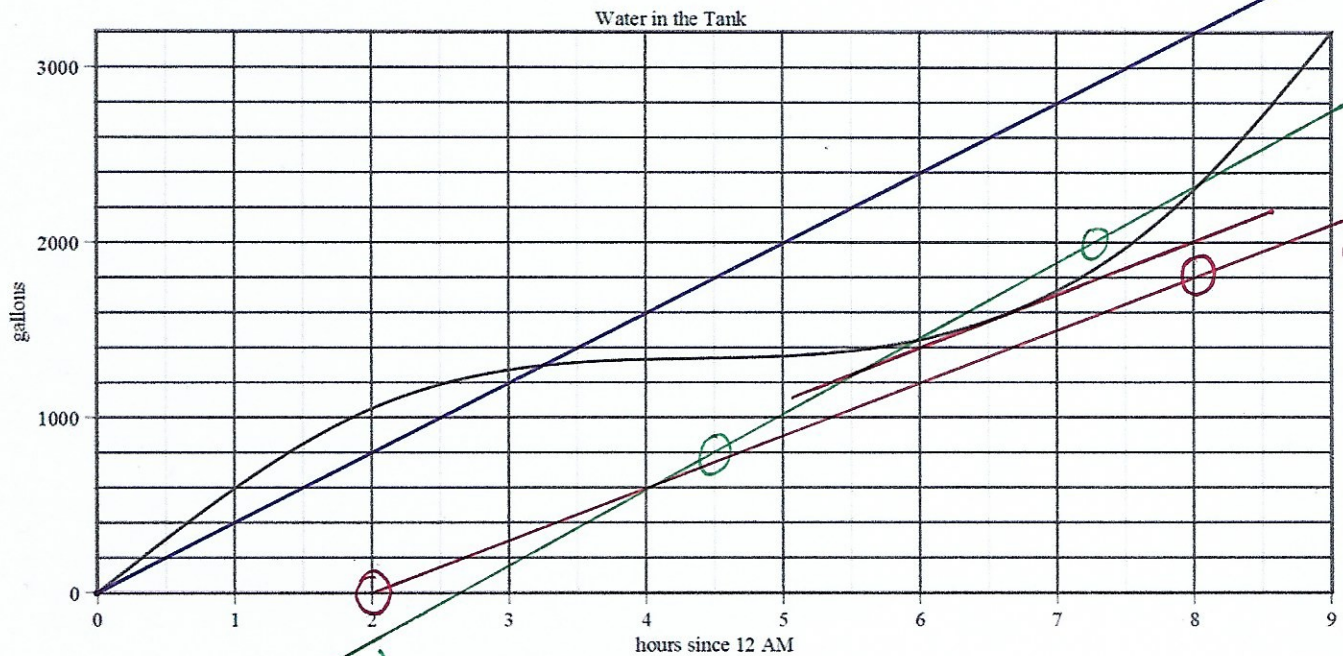


Solutions to Math III Wi '19 MT1

1. The following is a graph of the amount of water being pumped into a tank starting at 12 AM. Initially the tank is empty. Answer the following questions based on this graph. Label any lines you draw on the graph with the letter of the question so we can follow your work. Give UNITS with your answers.



- (a) (3 points) Find the Average Rate of Change of water being pumped into the tank in the 2 hour period starting at 6 AM. Round your answer to the nearest whole number.

$$\text{slope} = \frac{2000 - 800}{7.25 - 4.50} \approx 436 \text{ gallons per hour}$$

- (b) (2 points) Find a time when the Overall Rate of Change of water pumped into the tank is 400 gallons per hour. Give your answer like you would read it on a digital clock, e. g. 1:23 PM.

$$t \approx 3.2 \text{ hours} = 3 \text{ hours } 12 \text{ minutes so at about } 3:12 \text{ AM}$$

- (c) (2 points) At 2 AM, you start pumping water out of the tank at a constant rate of 300 gallons per hour. Graph the amount pumped out as a linear function of time on the picture above.

$$\text{slope} = 300 \text{ for run} = 6 \text{ rise will be } 1800$$

- (d) (3 points) Find the time when the amount of water in the tank (after you started the pump out described above) is minimum. How much water is in the tank at that time? Round your answer to the nearest gallon.

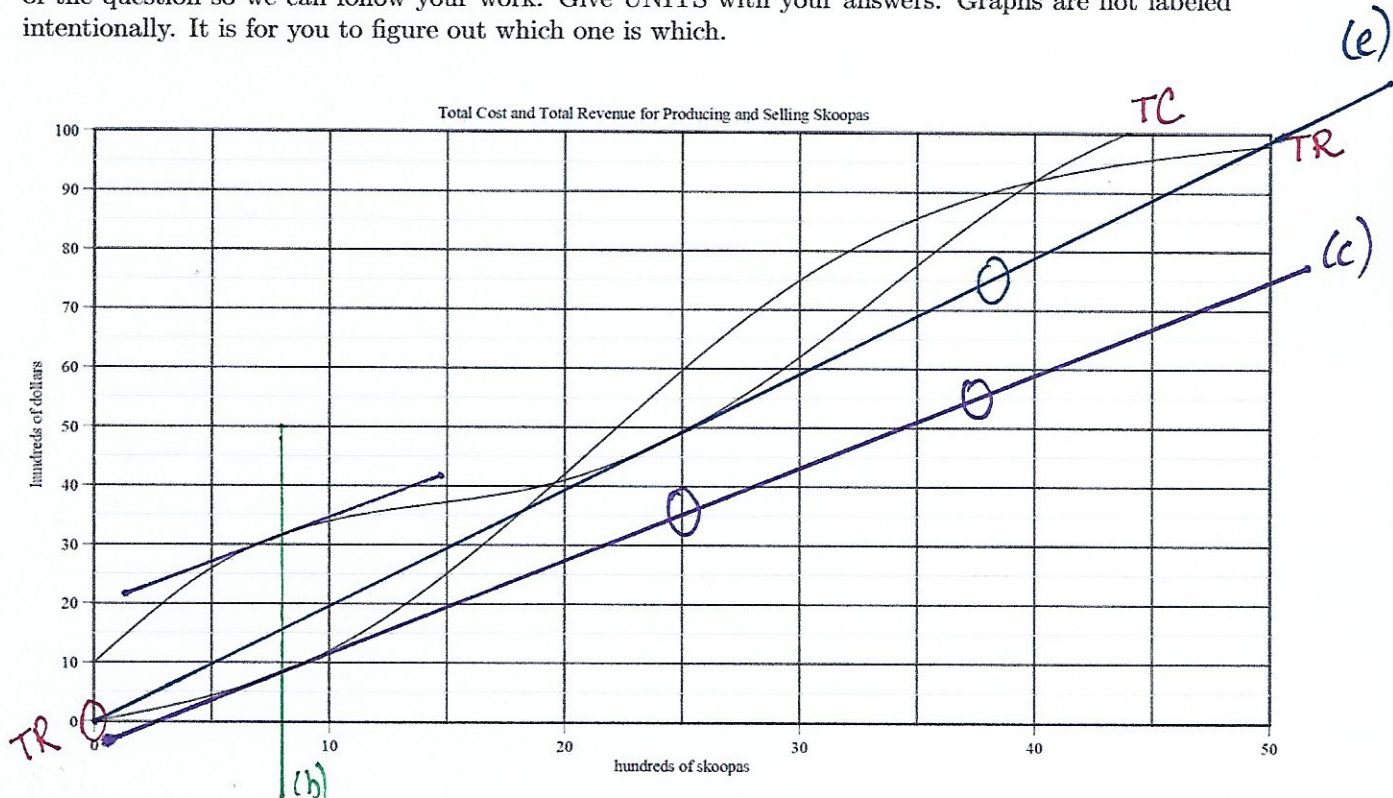
$$\text{at about } t \approx 6.6 \text{ hours}$$

$$\text{Amount in} \approx 1550 \text{ gallons}$$

$$\text{Amount out} = 300 \times (6.6 - 2) = 1380$$

$$\text{Difference} = 170 \text{ gallons}$$

2. The picture below shows the Total Cost and Total Revenue for producing and selling Skoopas. Answer the following questions based on this graph. Label any lines you draw on the graph with the letter of the question so we can follow your work. Give UNITS with your answers. Graphs are not labeled intentionally. It is for you to figure out which one is which.



- (a) (2 points) What is the Variable Cost at 3000 Skoopas? Give your answer to the nearest hundred dollars.

$$\begin{aligned} TC(30) &\approx 62 \text{ hundred} \\ FC &= 10 \text{ hundred} \\ VC(30) &= 62 - 10 = 52 \text{ hundred} \end{aligned}$$

- (b) (2 points) At what quantity do you have maximum loss? What is the maximum loss amount? Give your answer to the nearest hundred dollar.

$$\begin{aligned} &\text{At about 8 hundred Skoopas} \\ &\text{Around } 32 - 8 = 24 \text{ hundred Skoopas} \end{aligned}$$

- (c) (3 points) Compute Marginal Revenue at the quantity when you have maximum loss. Give your answer to the nearest cent.

$$\text{slope} \approx \frac{55 - 35}{87.5 - 25} = 1.60 \text{ dollars per Skoopas}$$

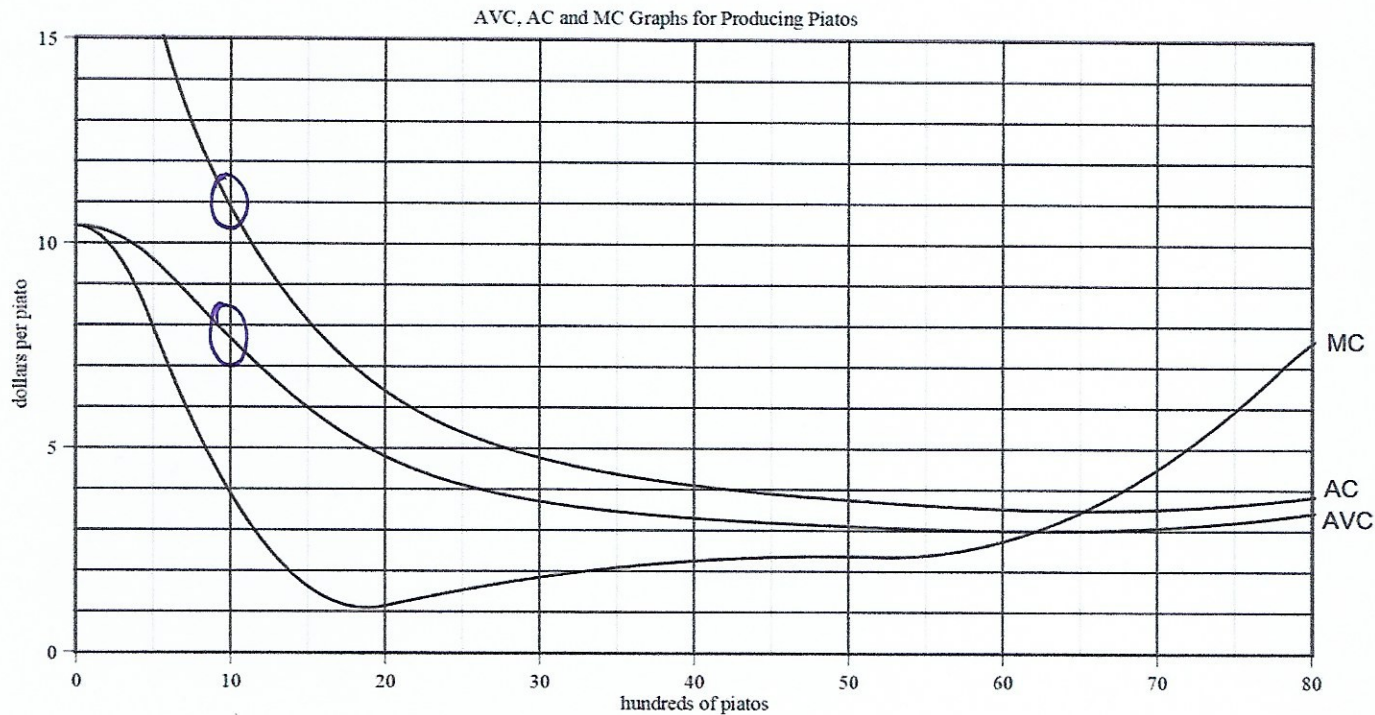
- (d) (1 point) Without any computation, what do you expect Marginal Revenue at the quantity where you have maximum loss?

This should be same as (c) if your vertical distance guess for (b) was good.

- (e) (3 points) What is the Breakeven Price? Give your answer to the nearest cent.

$$\text{slope} \approx \frac{75}{38} \approx 1.97 \text{ dollars per Skoopas}$$

3. The following are graphs of Average Cost, Marginal Cost and Average Variable Cost for producing and selling Piatos. Answer the following questions based on this graph. Label any lines you draw on the graph with the letter of the question so we can follow your work. Include UNITS with your answers.



- (a) (1 point) What is the Breakeven Price? Round your answer to the nearest 10 cent.

≈ 3.50 dollars per Piatos

- (b) (1 point) What is the Shutdown Price? Round your answer to the nearest 10 cent.

≈ 3.00 dollars per Piatos

- (c) (2 points) If you sell each Piatos for 4 dollars, at what quantity is the profit maximized? Round your answer to the nearest hundred Piatos.

At about 68 hundred Piatos

- (d) (4 points) What is the maximum profit? Round your answer to the nearest hundred dollar.

~~At about 68 hundred~~
 $TR(68) = 4 \times 68 = 272$ hundred dollars
 $TC(68) = 68 \times AC(68) \approx 68 \times 3.50 = 238$ hundred dollars
 $P(68) = 272 - 238 = 34$ hundred dollars

- (e) (3 points) Approximate the Fixed Cost. Round your answer to the nearest hundred dollar.

At any q , mine was $q = 10$
 $TC(10) = 10 \times AC(10) \approx 10 \times 10.9 = 109$ hundred dollars
 $VC(10) = 10 \times AVC(10) \approx 10 \times 7.7 = 77$ hundred dollars
 $\rightarrow FC = TC(10) - VC(10) = 109 - 77 = 32$ hundred dollars

4. The two parts of this question are not related.

(a) (4 points) Solve for y in terms of x :

$$\left(\frac{1}{x-2} + \frac{1}{y+1} = 3 \right) \cdot (y+1)$$
$$\left(\frac{y+1}{x-2} + 1 = 3(y+1) \right) (x-2)$$

$$y+1+x-2 = 3(y+1)(x-2)$$

$$y+x-1 = 3(xy+x-2y-2) = 3xy+3x-6y-6$$

$$y-3xy+6y = 3x-6-x+1$$

$$7y-3xy = 2x-5$$

$$(7-3x)y = 2x-5$$

$$\boxed{y = \frac{2x-5}{7-3x}}$$

(b) (4 points) Find the equation of the line through the points (1, 3) and (7, 6). Give your answer in the form $y = mx + b$.

$$\text{slope } m = \frac{6-3}{7-1} = \frac{3}{6} = \frac{1}{2}$$

$$\text{equation: } y-3 = \frac{1}{2}(x-1)$$

$$\text{simplify: } y = \frac{1}{2}x - \frac{1}{2} + 3$$

$$y = \frac{1}{2}x + \frac{5}{2} \quad \text{OR} \quad y = 0.5x + 2.5$$