STATEMENT OF PURPOSE

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I first began to get interested in mathematics the first quarter of my freshman year at the University of Washington, when I met the ε-δ formalism, everywhere-continuous yet nowhere-differentiable functions, and totally-disconnected yet uncountable subsets of $\mathbb{R}$ in my calculus class, and realized that the world of the real numbers was much richer and had much more structure than I had been led to believe. I had competed in math competitions through middle school, largely because I was good at that, but until entering the University of Washington through the Early Entrance Program, I thought of math as little more than a collection of interesting, yet essentially meaningless, number games.

At the end of that first quarter of my freshman year, I spent a few days asking around the applied math department — my naïve notion being that applied math would be more fun, less dry, and deeper than pure math — and found out that Bernard Deconinck was indeed willing to introduce a mathematically immature undergraduate knowing only calculus to the field of stability theory. After three months spent working through Roger Knobel's *An Introduction to the Mathematical Theory of Waves*, I spent the spring and summer examining the spectral stability of stationary solutions of the nonlinear Schrödinger equation and wound up with asymptotic formulae. It turned out that Prof. Deconinck and I had been unknowingly reproducing some work done by a British mathematician in the 1970s: a fine introduction, truly, to math research!

It was hard to keep my spirits from flagging, but after a basic course in multivariable calculus and real analysis, and a graduate sequence on ODEs and PDEs, I was able to approach the stability theory of integrable equations with a much more solid foundation. I worked actively with Prof. Deconinck until June 2009; the result of our labors is a general method for determining the spectra of stationary solutions of integrable equations. The first paper, titled "KdV Cnoidal Waves are Spectrally Stable", was published in the December 2009 issue of Discrete and Continuous Dynamical Systems; two more papers, one on each of the focusing and defocusing nonlinear Schrödinger equations, are in preparation. On the basis of this work, I was awarded a Davidson Fellowship in 2007, and a Goldwater Scholarship and Washington State Research Fellowship in 2008. My presentation of the KdV paper won one of four undergraduate poster prizes at the 2008 SIAM annual meeting.

While it has been wonderful to study stability spectra in such depth, my chief goal for my time as an undergraduate has been to get a taste of many flavors of math. With this in mind, I traveled to Moscow, Russia in August 2007 to study at the Independent University of Moscow for four months, supported by an AMS scholarship. There I took courses in the mathematics of computation, basic topology, ergodic theory, and algebraic geometry — as well as a Russian language course that sparked an intense interest in Russian literature, which eventually led me to declare a second major in Russian.

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What I found most interesting out of everything I studied in Moscow, however, was Gauss's arithmetic-geometric mean (AGM), a classical construction introduced to me by George Shabat, my algebraic geometry professor. Many years ago, Prof. Shabat had formulated a geometric construction of the AGM, and this construction yielded a natural generalization of the AGM to n variables. The motivation for this generalization was not purely academic: the classical AGM is a powerful tool for quickly computing elementary functions and elliptic integrals, so it seemed plausible that the generalized AGM could yield even faster, or more general, algorithms.

Over the course of the semester, I formalized the generalized AGM, and after returning to Seattle, I began exploring its intricacies with James Morrow, a professor in the math department whose 2007 REU I participated in. The most rewarding experience of my undergraduate math career was proving the real analyticity of the generalized AGM. I had been trying to prove this conjecture for six weeks, when I realized that Montel's theorem, which I had learned only two weeks before in the graduate complex analysis sequence, was the key to the proof — even though the statement of the conjecture had nothing to do with complex analysis! The connection was completely unexpected. Prof. Morrow and I are continuing work on the generalized AGM; we are learning about the Landen transformation in the hopes that this transformation will yield an integral representation of the generalized AGM, and a paper is in preparation.

Since January 2008, all of my math courses have been at the graduate level — this year, for instance, I am taking the graduate real analysis and algebra sequences — and I have loved the broad structure and complex interplay I have seen in all of this classical material. Under Freeman Dyson's classification of mathematicians as either frogs or birds, I would certainly self-identify as a bird. Yet despite the apparent breadth-over-depth approach to my undergraduate studies, I love the rush of nitty-gritty problem solving. In 2007, my team was named as one of 14 outstanding winners, out of 949 entrants, in the Mathematical Contest in Modeling, and I received a respectable 39 on last year's Putnam exam, an improvement over my score of 25 from the year before.

I have very little idea where subfield of math I will end up working in. That said, I am particularly interested in studying the analysis of PDEs and dynamical systems. Princeton, then, would be an ideal place for me to do my graduate studies: the opportunity to work with people like Sergiu Klainerman, Weinan E, and Elias Stein would be incredible, and I know the idyllic beauty of the Princeton campus from having spent three weeks there in the summer analysis program.