INSTRUCTIONS — READ THIS NOW
• This test has 5 problems on 6 pages (counting this cover) worth a total of 100 points. Look over your test package right now. If you find any missing pages or problems please ask me for another test booklet.
• Write your name and your NetID right now.
• Show your work. To receive full credit, your answers must be neatly written and logically organized. If you need more space, write on the back side of the preceding sheet, but be sure to clearly label your work.
• This is a 50 minute test. Students taking this test were allowed to use a scientific calculator (not a graphing calculator) and bring consult one page of hand-written notes (front and back), on standard size paper.
• You must put a [BOX] around your final answer. You DO NOT need to simplify your answers except when explicitly asked to.
• TURN OFF YOUR CELL PHONE.

OFFICIAL USE ONLY

1. _____
2. _____
3. _____
4. _____
5. _____

Total:_____
Problem 1 (20 points): Find the solution to the initial value problem

\[ \frac{dy}{dx} = \frac{3x^2}{(2 + x^3)y}, \quad y(-1) = -2 \]

Make sure you write your solutions for \( y \) as an explicit function of \( x \).
Problem 2 (20 points): Solve the initial value problem

\[ ty' + 2y = e^{2t}, \quad y(1) = 0. \]

Make sure you write your solution for \( y \) as an explicit function of \( x \).
Problem 3 (20 points): Water is pouring into a tank at a rate proportional to the square root of the amount of water present in the tank at that instant. At time $t = 0$ minutes, there is 1 gallon of water in the tank, and at time $t = 1$ minute, there are 4 gallons in the tank. How many gallons of water are in the tank at time $t = 3$ minutes?
Problem 4 (20 points): Consider the autonomous differential equation

\[ y' = y^3 - 2y. \]

(a) Find all equilibrium solutions to the equation.

(b) In the space below, sketch the direction field for the equation so that all equilibrium solutions appear in your sketch. Be sure to label your axes.

(c) For each equilibrium solution in part (a), say whether it is stable, unstable, or semi-stable, briefly justifying each answer.
Problem 5 (20 points): Consider the initial value problem
\[ \frac{dy}{dx} = t, \quad y(0) = 1. \]

(a) Use Euler’s method, with step size \( h = 0.1 \) and starting at \( t_0 = 0 \), to approximate the value of the solution at \( t = 0.4 \).

(b) As a check, solve the equation above to get the exact value of the solution at \( t = 0.4 \).